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WP-083-2019

# Economic Modeling of Climate Risks

Research & Macro Strategy

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## Abstract

Climate change is a subject that has been largely addressed from both macroeconomic and energetic standpoints. Integration of climate variables and natural capital into the traditional economic framework can appear conflicting with the notion of infinitely growing economies exploiting finite resources, which questions the sustainability of neoclassic economic growth models. Moreover, the temporal dimension is of paramount importance and the integration of inter-temporal utility is not a trivial issue. The construction of complex general equilibrium models is a way to model the response of economic systems to shocks. Their use is somewhat limited because of their lack of transparency, computational scaling issues and non-equal attitudes toward uncertainty. If they are properly calibrated to model scenarios of interest, these models can however constitute an additional module for assisting short- and medium-term decisions. The dynamic integrated climate economy (DICE) seminal model of William Nordhaus allows to set an optimal global control trajectory with respect to a set of constraints and assumptions. Similar and more sophisticated macroeconomic models can provide the optimal allocation with respect to long-term constraints.

The complexity of the academic literature might have clouded a rather simple question. Will we efficiently reduce the negatives implied by our economic activity or face the consequences? Consequently, the two aspects an investor wishes to assess is to what extent his portfolio contributes to the reduction of social and environmental negatives, and how it contributes to the improvement of global resiliency. The first dimension can be approached with integrated assessment models (IAMs) similar to the DICE, with clear and fair expression of trajectories required from each sector and region.

The remaining pitfalls are to set commonly accepted abatement cost curves and to obtain full disclosure of the research and development investment dedicated to climate change and to set issuer-specific deviation from the optimal path. Similarly, commonly accepted accounting techniques are required to meet this goal. Regarding the second dimension and the question of adaptation, there is a lack of behavioral modeling and indicators of the resiliency dimension where huge uncertainty remains and will not be dealt without the consideration of the social dimension.

**Keywords:** Economic modeling, climate change, scenarios, energy, general equilibrium, technical change, optimal policy.

**JEL classification:** E00, Q5, O33.

## Acknowledgement

* *I am very grateful to Frederic Lepetit, Thierry Roncalli, Takaya Sekine and Peter Tankov for their helpful comments.*

## About the authors

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Théo Le Guenedal joined the Quantitative Research team of Amundi in December 2018 after his internship dedicated to the performance of ESG investing in the equity market. He is currently working on a research project "Integration of ESG Factors and Climate Risks in Asset Allocation Strategies" .

Théo graduated from Ecole Centrale Marseille with a specialization in Mathematics, Management, Economics and Finance. He also holds a Master degree in Mathematics and Applications from Aix-Marseille University. Prior to that, Théo worked on modeling and mapping renewable energy supply for BG consulting engineers. In 2017, Théo was awarded the postgraduate diploma "Engineers for Smart Cities" from the Mediterranean Institute of Risk, Environment and Sustainable Development and a Master degree in Economic Management from the School of Economics and Business of Nice Sophia Antipolis University.

# 1 Introduction

Extreme weather events and natural disasters are now considered the most likely and serious economic risks[1]. If the notion of climate risks can first appear intuitive, measuring these risks is in fact still a quantitative puzzle. Since the great financial crisis, a particular attention has been dedicated to risk transmission channels to financial stability. Mark Carney (2018) identified three forms of transmission channels from climate risks. First, the *physical risks* stemming from the increased frequency and severity of climate and weather events causing physical capital destruction. The *liability risks* that arise from actors seeking compensation from those they hold responsible for the climate related losses they suffer. Finally, the *transition risks* that arise from the sudden shift toward low-carbon economy. In practice, issuers and asset managers generally consider two main ways climate change can affect their activity[2]. On the one hand, the *climate risks* gather actors' physical risks and the adaptive capacity of their business model. To some extent, these risks are related to the capacity of a actor to adapt in a changing landscape. Following this definition, a car manufacturer faces a climate risks if his facilities are located on a sensitive area but also if he appears unprepared to shift his business given the current and expected changes required in the transportation sector. In other words, these risks are default risks that can be qualitatively represented as the answer to the question: can the actor subsist physically and economically if the average temperature was to rise? On the other hand, *carbon risks* are more closely related to the carbon intensity of their activity and often associated with transition or regulatory risks (Rose, 2014). In simple terms, these risks are the answer to the question: what would the actor have to pay if the optimal tax was implemented and would its business still be profitable?

Regardless of the precise definition of these risks, the estimates generally come from integrated economic models that we must fully understand in order to make good use of the results. The purpose of these models is to estimate the evolution of the global economy under a range of assumptions given growing uncertainty. In a forward-looking environment, it can be misleading to assess future outcomes on historical data and this is the reason why these mathematical frameworks, called integrate assessment models (IAMs), are developed. They allow climate risks to be quantified and provide information on the optimal mitigation or adaptation policies to prevent them. Climate change being a global phenomenon, IAMs are generally built on the basis of macroeconomic modeling.

The macroeconomics of climate change lie on six important dimensions: growth, time horizon, geographic breakdown, complexity and trade-flow (regions and sectors), damage modeling due to uncertainty and resiliency or adaptive capacity. The first two dimensions question the interaction between climate change and standard growth theory in the long-run. The next two respectively translate the difficulties of having a representative model allowing interactions between agents and to estimate the spatial distribution of damages. Modeling and quantifying expected damages is a challenge in itself. For instance, modelers must distinguish direct impacts, second round effects (Battiston *et al.*, 2017) and extreme events with their respective magnitude and likelihood without data to back up their projections. Finally, assessing either the resiliency or adaptive capacity of agents evolving in a changing landscape is by no means trivial. Above all, the data required to

[1] World Economic Forum Risk Report 2017: http://www3.weforum.org/docs/GRR17_Report_web.pdf.

[2] Considering that liability risks mainly result from the diffusion of the physical risks through the insurance, legal or financial system.

track the macroeconomic dynamics of climate change lie with microeconomic indicators which ends-up in the conflicting top-down vs. bottom-up methodological debate.

In neoclassical Keynesian economics, the internalization of externalities occurs through government or central banks actions. Therefore, numerous models tracking the optimal carbon tax have been developed by practitioners. The purpose of these models is to put a price on emissions, defining thus the social cost of carbon. In addition to the DICE model described in this paper, we can for example introduce PAGE[3] (Hope, 2006; Alberth and Hope, 2007), which is a model which considers eight regions and four damage components: economic damages, non-economic damages, sea level rise and discontinuities. This model was used by Nicolas Stern in *the Stern Review* (2006), the famous report advocating for immediate action. The FUND[4] (Anthoff and Tol, 2009 and 2012) model is an example of a partial equilibrium model that can be used on case studies to assess the impact of specific climate events on GDP for instance (Narita *et al.*, 2009). This model considers sixteen regions and different sources of climate damages: health, sea-level rise, agriculture etc. Another famous example is the MERGE[5] (Manne and Richels, 2005) which was originally introduced by Manne *et al.* (1995). It describes global economy-climate interactions between nine specific regions and uses Negishi weights to balance interregional trade-flows (Stanton, 2009). These models are used by governments to determine the social cost of carbon. We consider that they can be used in the field of finance to assess assets exposure to carbon risks.

The concept of integrating climate variables was extended to the macroeconomic real business cycle, where the money supply follows a stochastic process. In these conditions, one can observe the resulting equilibrium of the variables of interest given a set of assumptions. These models are therefore called dynamic stochastic general equilibrium (DSGE) models. In simple terms, they represent time varying processes, embedding to some extent stochastic uncertainties, related to financial instruments, modeled in order to make some projections and to find plausible equilibria. In the case of climate change, the variables of interest will therefore be the common economic ones (prices, production and demand for goods) plus some endogenous variables, such as temperatures or sea levels, that have been ignored so far despite their economic outcomes. Moreover, the uncertainty related to climate change and social stability can also justify to some extent stochastic modeling. This type of tools have a modular structure and can provide complementary information on transition pathways. Moreover, they can be fitted on input-output matrices to match observed time-series or allow endogenous technical changes and substitutions. These advanced models are powerful tools to assist decision-making in the field of asset management as long as managers are well aware of their limits.

This paper aims to provide an extended review of the literature related to IAMs and more broadly on climate economy. It is organized as follows. First, we reiterate the theoretical macroeconomic fundamental notions that enable IAMs to be developed. Then, we provide a detailed review of the seminal model of Nordhaus, the dynamic integrate climate economy (DICE) model. Third, we present a literature survey of the modeling standards of the main variables and phenomena of interest. In this section, we question model outputs with respect to the underlying uncertainties and their failure to represent the world's complexity. Despite the imperfection of these frameworks, they are the only tools available to make out-of-sample estimates. Conse-

[3]PAGE is the acronym for *policy analysis of the greenhouse effect.*

[4]FUND is the acronym for *framework for uncertainty, negotiation and distribution.*

[5]MERGE is the acronym for *model for evaluating regional and global effects of GHG reduction policies.*

quently, our view is that extending these models to model endogenous growth rates and include the financial market and particularly investor's range of different beliefs could be a question to address. Therefore, further disclosure and data support are required to allow asset managers to track portfolio-specific abatement costs relative to emissions induced and to implement, for instance, robust constrained portfolio alignment strategies. More generally, to better integrate both short- and long-term objectives, financial pricing models must encompass multiple modules to adopt optimal strategies in the changing landscape.

# 2 The origin of climate economy

Climate integrated economy literally represents the incorporation of climatic constraints into the traditional economic framework. Commonly, the economic growth engine is based on an exogenous labor force or population, a productivity factor and the idea according to which capital is split between investment and consumption. Integrating climate into this framework suggests that the increasing temperatures will, one way or another, have an impact on growth. To translate this, one can introduce hypothetical global damages, which requires strong assumptions for a global model, and are even less clear looking at atomistic agents. Prior to that, we must introduce the academic concepts for representing economic growth equilibrium. Once the economic foundation is laid, we provide a review of Nordhaus optimization model for policy making, which was the first to introduce a climate module in the economic framework.

## 2.1 Economic growth modeling

The extended literature on climate economy is based on the famous and traditional Solow (1956) growth model[6]. Before introducing environmental constraints, we give an overview of this model, and of the infinite-horizon and overlapping generations modeling standards. The thorough theoretical concepts are precisely developed by Romer (2006). The third part aims to introduce the notion of decentralized equilibrium, allowing the markets' complexity to be estimated.

### 2.1.1 The Solow growth model

The Solow model is based on a standard function defining the production of a single good $Y(t)$:

$$Y(t) = F(K(t), A(t)L(t)) \tag{1}$$

where $K(t)$ is the capital input, $A(t)$ the *knowledge factor*[7] and $L(t)$ the labor all varying over time $t$.

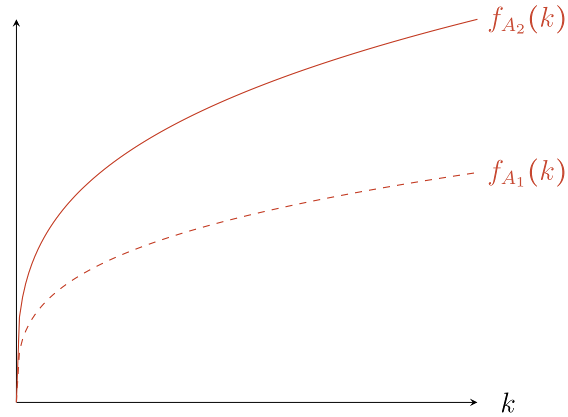
Graphically, Figure 1 shows that increasing the productivity factor $A_2 > A_1$ significantly shifts up the production. The question raised would be how to capture disruption in this knowledge factor that affects production[8]. In the Solow model, production functions have constant *returns*

[6] Sometimes called the Solow-Swan (1956) model.

[7] See knowledge factor in the glossary.

[8] The question of endogenous technical change (ETC) will be addressed in Section 3.2 on page 38.

Figure 1: Influence of knowledge factor in the Solow model




*to scale*[9], meaning that they meet the condition:

$$F(cK, cAL) = cF(K, AL) \quad \forall c \geqslant 0 \tag{2}$$

For a set of inputs $I$, for instance $I = \{A, L, K\}$, the production function $F : I \to O$ must also respect Inada conditions (Inada, 1964), that gives, in the general case:

1. $F(0) = 0$
2. $\partial F(x)/\partial x_i > 0 \quad \forall x_i \in I$
3. $\partial^2 F(x)/\partial^2 x_i < 0 \quad \forall x_i \in I$
4. $\lim_{x_i \to +\infty} \partial F(x)/\partial x_i = 0$
5. $\lim_{x_i \to 0} \partial F(x)/\partial x_i = +\infty$

Conditions (2) and (3) imply that marginal production with the capital is positive but decreasing while capital rises. Altogether, these conditions give production functions with logarithmic shapes (Figure 1). The constant returns condition (2) enables us to write:

$$F\left(\frac{K(t)}{A(t)\,L(t)}, 1\right) = \frac{1}{A(t)L(t)} F(K(t), A(t)L(t))$$

We set $k(t) = \dfrac{K(t)}{A(t)\,L(t)}$ to be the capital per unit of effective labor. Then, the output per unit of labor is given by $y(t) = \dfrac{1}{A(t)\,L(t)} F(K(t), A(t)\,L(t))$ and $f(k) = F(k, 1)$, so we can rewrite the production function with the following reduced form:

$$y(t) = f(k(t))$$

[9] See return to scale in the glossary.

The exogenous growth of labor and knowledge are generally represented by exponential functions with given growth rates:

$$\begin{aligned} A(t) &= A_0 e^{g_A t} \\ L(t) &= L_0 e^{g_L t} \end{aligned}$$

where $g_A$ and $g_L$ are respectively the growth rates for productivity and labor initiated at $A_0$ and $L_0$. In this model, the output can be saved for future generations, therefore it is split between consumption and investment. This important relationship is called *law of motion* for capital, we have:

$$\frac{\mathrm{d}K(t)}{\mathrm{d}t} = sY(t) - \delta_K K(t)$$

where $s$ is the saving rate and $\delta_K$ the depreciation of capital. This model is highly simplified and does not take into account governments, or fluctuation in employment, while the rates $s$, $\delta_K$, $g_L$, $g_A$ are constant and independent. Moreover, there is a single product. The key equation of the Solow model is given by deriving $k(t)$:

$$\begin{aligned} \frac{\mathrm{d}(K(t)/A(t)L(t))}{\mathrm{d}t} &= \frac{\mathrm{d}k(t)}{\mathrm{d}t} \\ &= sf(k(t)) - (g_L + g_A + \delta_K)k(t) \end{aligned}$$

A specific example, used in Nordhaus model, is the Cobb-Douglas production function. This function has constant elasticity of substitution and meet all Inada conditions.

- In the case of Harrod-neutral, we have: $Y(t) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha}$;
- In the case of Hicks-neutral, we have: $Y(t) = A_{\text{TFP}}(t)K(t)^{\alpha}L(t)^{1-\alpha}$.

Here, $A_{\text{TFP}}(t)$ is the total factor of productivity (TFP) and $\alpha$ is the share of capital in revenue defining the input output elasticity. For instance, if $\alpha = 0.2$, an increase of 1% in capital usage would lead to approximately a 0.2% increase in output. Cobb Douglas presents the advantage to be the same regardless of the introduction of the technological progress with the identity: $A_{\text{TFP}}(t) = A^{1-\alpha}(t)$. We recall the expression at the equilibrium:

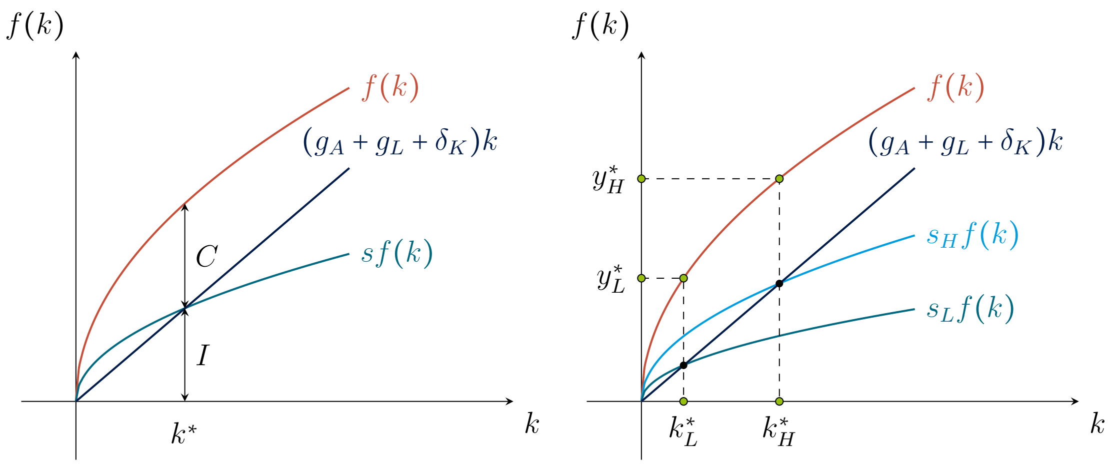
$$sf(k^{\star}(t)) = (g_L + g_A + \delta_K)k^{\star}(t)$$

Using the Harrod-neutral specification, leading to the simplified expression $f(k) = k^{\alpha}$, gives the famous equilibrium:

$$k^{\star} = \left(\frac{s}{g_L + g_A + \delta_K}\right)^{\frac{1}{1-\alpha}}$$

Figure 2 shows that increasing the saving rate, $s_H > s_L$, increase investment, below the intersection, and reduces consumption, between intersection and production output $f(k)$, which lead to another equilibrium. Phelps (1961) golden rule saving rates, or golden rule of accumulation, is the expression of the saving rate that satisfies the equilibrium, and more specifically that maximizes consumption over time and between generations. It expresses in simple terms, the fair rate of savings to preserve future generations:

Figure 2: Golden rule of savings in the Solow model




*"Each generation in a boundless golden age*[10] *of natural growth will prefer the same investment ratio, which is to say the same natural growth path"* (Phelps, 1961, page 640).

The Solow version states that the capital/output ratio depends only on the saving rates, growth and depreciation rate:

$$\frac{K(t)}{Y(t)} = \frac{s}{g_L + g_A + \delta_K}$$

Mankiw-Romer-Weil (1992) version of model proposed to take into account '*human capital*' accumulation. This new version of the Solow model presents the output as follow:

$$Y(t) = H(t)^{\beta} K(t)^{\alpha} (A(t) L(t))^{1-\alpha-\beta}$$

where $H(t)$ is the human capital and $\beta$ is the elasticity human capital–effective labor. Assuming the same depreciation rate $\delta_H = \delta_K$ for human and physical capital, and admitting that the savings can be split up between human and physical capital such as $s = s_H + s_K$, there is convergence toward the following equilibrium:

$$k^{\star} = \left(\frac{s_K^{1-\beta} s_H^{\beta}}{g_L + g_A + \delta_K}\right)^{\frac{1}{1-\alpha-\beta}} \quad \text{and} \quad h^{\star} = \left(\frac{s_K^{\alpha} s_H^{1-\alpha}}{g_L + g_A + \delta_K}\right)^{\frac{1}{1-\alpha-\beta}}$$

Presented this way, it is both natural and intuitive to extend this concept to the '*natural capital*' which could be represented by the *carbon budget*[11] in models estimating global warming. Note however, that regardless of the quantity accumulated, this model leads to static equilibrium and the time dimension is absent. More importantly, long run growth derives mostly from TFP growth[12], an exogenous variable about which the model is entirely silent. Accounting growth

[10] *"By a golden age I shall mean a dynamic equilibrium in which output and capital grow exponentially at the same rate so that the capital-output ratio is stationary over time"* (Phelps, 1961, page 639).

[11] See carbon budget in the glossary.

[12] See Figure 1 on page 10.

methods exist but are subject to numerous pitfalls due to their dynamic aspect and the perfect competition assumption under which Cobb Douglas production function holds.

The question becomes how to better represent the natural capital in a *neoclassic*[13] model. Indeed, resources or the planet's global state are not included in this basic approach. Malthus (1798) theory already questioned the adequacy between the exponential growth of the population and food supply. His theory can be extended to other limited resources or, more generally, to any irreplaceable input consumed faster than produced[14]. To find the most appropriate answer from policy makers or investors, we must analyze how to best model climate impact on the economy. Is it using the traditional Solow model where the output is penalized by a climate loss coefficient $\Omega_{\text{climate}}$? If yes, how do we mathematically assess damages, potentially endogenous to the output, and compute their effects on the production? Or should we consider a decreasing stock of the natural capital available in a traditional accumulation approach? For instance, the latter option would lead to the following equation:

$$Y(t) = K(t)^{\alpha} \left(A(t) L(t)\right)^{1-\alpha-\vartheta} N_c(t)^{\vartheta} \tag{3}$$

where $\alpha > 0$ is the elasticity capital–effective labor, and $\vartheta > 0$ the elasticity of natural resources–effective labor and admitting that $\alpha + \vartheta < 1$. The natural capital[15] $N_c$, which is decreasing, which gives mathematically $\dfrac{\mathrm{d}N_c(t)}{\mathrm{d}t} = -\delta_{N_c} N_c(t)$, $\delta_{N_c}$ being a positive constant[16]. Differentiating now Equation (3) and after taking the logarithms of each sides, we can obtain the growth rate for the output on the balanced growth path[17] $g_Y^{\star}$ (Romer, 2006):

$$\begin{aligned} g_Y^{\star} &= \frac{(1-\alpha-\vartheta)(g_L+g_A) - \delta_{N_c}\vartheta}{1-\alpha} \\ &= g_L + g_A - \frac{(g_L + g_A + \delta_{N_c})}{1-\alpha}\vartheta \end{aligned}$$

This illustrative example gives an idea of the penalization growth factor implied by the introducing of a decreasing environmental capital. However, consumption of exhaustible assets and resources through static-equilibrium type of economic theory can be misguided:

> "*The static-equilibrium type of economic theory which is now so well developed is plainly inadequate for an industry in which the indefinite maintenance of a steady rate of production is a physical impossibility, and which is therefore bound to decline*" (Hotteling, 1931, page 138-139)[18].

[13] See neoclassic economics in the glossary.

[14] In this case, carbon comes as a negative capital, which is emitted faster than absorbed.

[15] We could have defined a function for land use and another for resources similarly to Romer (2006).

[16] Some contemporary theories, like the Green Paradox (Sinn, 2012), would imply that $\delta_{N_c}$ is in fact an increasing function of the time, as instead of speculating on resources that will become more expensive because their rarefaction, owners tend to get rid of them before they become stranded.

[17] This result is based on the hypothesis that $g_K = g_Y$. We also use a labor augmenting factor $A(t)$ which is not the case in most models using the $A_{\text{TFP}}$ Hicks-neutral version.

[18] However, following Hotteling's principles we should observe in theory that, the rate at which the price of non-renewable resources increases tracks the real interest rate. In practice, it does not.

Not to question the entire neoclassic approach, describing the dynamics of an infinitely growing system in a finite universe, we can to some extent admit that looking for static growth equilibrium is not adapted to investigate climate potential damages or to determine exhaustible resources prices. Therefore, in his original seminal paper, William Nordhaus chose to introduce a damage coefficient, a function of the temperature, reducing the net output.

### 2.1.2 Infinite-horizon and overlapping-generations

In long-term issues, the problem is how to model global preferences through time. This issue was introduced by Mark Carney[19] (2018) as the '*tragedy of horizon*' of climate change. The static golden rule has shown its limits when it comes to represent global growth with limited resources. Therefore, we must use a modeling tool that enables us to integrate time preference. Utility functions are used in economics to model preferences and rank alternatives according to their utility to the agent they represent. Most consider a welfare function as the aggregated social utility, a function of consumption per capita. In these models every household have therefore the same utility; we talk about '*identical infinitely lived households*'. This type of representation is often used to obtain the best policy in the centralized case, where the social planner looks for an optimal trajectory. Note however, that these models are subject to criticism as in the '*real world*' agents are subject to highly inegalitarian revenues, implying thus different pattern of consumption[20] between households, and with high and obvious preference for the present coming as a standard in most economic and financial models (time value of money).

The economy is generally modeled using a large number of identical households. Each of them is said to provide one unit of labor $L(t)$ which follows the growth rate $g_L$. The inter-temporal utility function in the traditional Ramsey (1928) model is:

$$U = \int_t^{\infty} e^{-\rho t} L(t)\, u\left(\frac{C(t)}{L(t)}\right) \mathrm{d}t$$

where $\rho$ is the discount rate that represents time preference, $C(t)$ is the consumption and the $u$ is the utility function. Let $c(t) = \dfrac{C(t)}{L(t)}$ be the consumption per capita. The utility function used is generally the constant relative risk aversion (CRRA):

$$u(c(t)) = \begin{cases} \dfrac{1}{1-\theta} c(t)^{1-\theta} & \text{if } \theta > 0, \quad \theta \neq 1 \\ \ln c(t) & \text{if } \theta = 1 \end{cases}$$

[19] "*Climate change is a tragedy of the horizon which imposes a cost on future generations that the current one has no direct incentive to fix*" (Mark Carney, Governor of the Bank of England, speech 6 April 2018, International Climate Risk Conference for Supervisors, De Nederlandsche Bank, Amsterdam).

[20] One could indeed wonder how to model a system based on, if not digging, inequalities, with a single agent scalable to the entire population. If the imperfections of economic models are admitted and are, to some extent, what makes them models, the concrete means to act against climate change might be revealed by entering these inequalities in this model. The simple underlying idea is that it would make sense to expect more from the ones that can do more. This type of distinction has been partly done at a regional level by a later regional version of the DICE (see Section 2.2.3 on page 32).

where the parameter $\theta$ measures the degree of relative risk aversion. In this case, $\theta$ also represents the willingness to shift consumption between periods. Another extension to this model was to propose an overlapping-generations model (Diamond, 1965). The utility function is divided in two generations respectively consuming $c_1(t)$ and $c_2(t)$. The utility for the representative household at $t$ becomes:

$$u(c_1(t), c_2(t)) = \frac{c_1(t)^{1-\theta}}{1-\theta} + \left(\frac{1}{1+\rho}\right)\frac{c_2(t+1)^{1-\theta}}{1-\theta}$$

where $\theta > 0$ is the risk aversion, $\rho > -1$ and where the consumptions of the two generations are subject to economic dynamics. This type of operation, splitting the overall consumption, is usually aimed at testing specific products, measures or policies. It was introduced by Diamond (1965) to assess national debt and government action but can be extended to other simulations. For instance, if we wish to estimate the consequences of a Neo-Malthusian scenario, including birth control, we must take this type of function into account. Indeed, the current pension system might fail to match the consumption of the older generation under a certain growth rate. Many other applications could be found for instance for modeling employment or optimal duration of schooling.

Ramsey's model applies to deterministic cases. The introduction of random risk requires tools capable of taking into account uncertainties. In the event of future hazard, the function of Epstein-Zin (1991) is commonly used to assess inter-temporal utility. Indeed, this function was introduced to allow a discrete differentiation to inter-generational preferences. The Epstein-Zin utility distinguishes risk aversion and elasticity of inter-temporal substitution (EIS), while CRRA considers that agent's willingness to substitute consumption across states of nature is the same as their willingness to substitute consumption over time. This utility function therefore makes the distinction between the two parameters and follows:

$$u_{\mathrm{EZ}}(c(t)) = \left((1-\beta_T)c(t)^{\rho} + \beta_T\left(\mathbb{E}_t\left[u_{\mathrm{EZ}}(c(t+1))^{1-\theta}\right]\right)^{\frac{\rho}{1-\theta}}\right)^{\frac{1}{\rho}}$$

where $\frac{1}{\beta_T} - 1$ is the marginal rate of time preference, $\frac{1}{1-\rho}$ is the elasticity of inter-temporal substitution and $\theta$ is the risk aversion parameter[21]. This function is recursively defined and we note that maximizing the utility at $t$ requires an idea of the distribution of consumption at $t+1$. This notion of future utility is the certainty equivalent of future lifetime utility, translated by the expected value given the information available at time $t$ (Daniel *et al.*, 2018). If the uncertainty at $t_\infty$ is high, this type of function channels the aversion toward temporal uncertainty and potentially suggests prompt actions to preserve future utility. Consequently, Daniel *et al.* (2017), using this framework to assess the social cost of carbon, found a higher price with EZ-climate[22] than with traditional CRRA, especially when climate uncertainty rises:

[21] Ha-Duong and Treich (2004) indeed showed that climate economy integrated models based on inter-temporal expected utility maximization setting $\theta = \rho$, in other terms, setting the same elasticity of substitution between states and times, "*may misinterpret the sensitivity of the climate policy to risk-aversion*" (Ha-Duong and Treich, 2004, page 1). In their paper they compared the sensitivity of the two parameters of interest, namely risk aversion and the resistance to substitution.

[22] Note that we can find similar works comparing preference for the present utility to power utility with risk aversion (CRRA) for instance Bansal *et al.* (2016).

> "*The optimal carbon price may, in fact, be high today, declining over time [...] The less certain we are about the climate risks facing us in future states of the world, the higher the optimal price on carbon today*" (Daniel et al., 2018, page 43).

There are multiple ways to model inter-temporal utilities and specifications for these functions depending on the problem to solve. The behavioral issues implied by temporal uncertainty are, however, poorly represented and this theoretical framework ignores the complexity of the '*tragedy of the commons*'[23] system archetype of climate change for example. Another concrete example to picture this vicious cycle could be be through the answer provided by the fossil fuel supply side, boosting their sales before becoming stranded. Projecting future taxes and a significant decrease in demand for their products, the primary incentive of owners of fossil resources is to lower prices, and doing so, increase global emissions. This phenomenon, worsening the global situation and referred to as the green paradox (Sinn, 2012), is most likely anecdotal but reflects the mismatch between the optimal inter-temporal behavior and the one observed on agents interacting on the market place. To go further in the modeling of contextual behavior, immediate responses, cascading effects and their implication on prices, we must consider more complex representations. The decentralized computable general equilibrium (CGE) models are good candidates to approach these rather short- and medium-term responses.

### 2.1.3 Decentralized general equilibrium

So far, we have considered utility functions relative to centralized or dictatorial allocation processes, where every choice at each step is made by a dictator-like decision maker, a so-called social planner. Introducing a decentralized allocation process allows us to extend the modeling landscape to the real business cycle of firms in competitive markets. In other words, these dynamic models aim to best reproduce the reality to find the most expected equilibrium in a situation where there is no almighty social planner and where each agent will maximize its own utility using the information in its possession, or in other words, given its *rational expectations*[24].

In this extension of the Solow model, the Firm $i$ receives capital from households in exchange for rental rate. In other words, it is owned by them. Each firm can be seen as a production unit of a single or multi-dimensional output, the reduced production function is noted $f$. Their production constrained by both domestic and foreign market demand, which fixes prices $p(t)$. The Firm seeks to maximize profits given $k_i(t)$ and $L_i(t)$ that are respectively the amounts of capital and labor that Firm $i$ employs at time $t$. The first order conditions for an interior solution allow us to determine the optimal interest rate and wage. The interest rate therefore follows:

$$r_i(t) = \frac{\mathrm{d}f_i(k_i(t))}{\mathrm{d}t}$$

and the wage per unit of effective labor follows:

$$w_i(t) = f_i(k_i(t)) - k_i(t)\frac{\mathrm{d}f_i(k_i(t))}{\mathrm{d}t}$$

[23]See tragedy of the commons in the glossary.
[24]See rational expectations in the glossary.

Each representative Firm $i \in \mathfrak{S}$ in the market maximizes its utility of profits given by:

$$\begin{aligned}\Pi_i &= f_i(k_i) - r_i(t)\,k_i(t) - w_i(t)\,L_i(t)\\ &= y_i - r_i(t)\,k_i(t) - w_i(t)\,L_i(t)\end{aligned}$$

where $y_i(t)$ is the production of Firm $i$, $w_i(t)$ is its wage, $r_i(t)$ is the interest rate, $L_i(t)$ and $k_i(t)$ the labor and capital involved in the production. This relationship, added to the maximization of household utility and market clearing condition[25], are sufficient to set the mathematical framework in which real business cycles can be tested. Under these conditions the firm's maximum profit is exactly zero and the prices defined by market demand do not intervene. In this case, this set of equations defines the zero profits condition. Further specifications embedding prices for products $y_i(t)$ can allow us to estimate profit distributions for firms. If the choice is made to embed a stochastic definition for prices, this framework becomes a dynamic stochastic general equilibrium (DSGE) model that allows us to simulate the convergence of the market with uncertainty. Particularly, this dynamic framework could allow us to shock and stress-test, to assess the economic system response to an identified event, if causalities, feedbacks and transition pathways have been properly defined.

In computable general equilibrium (CGE) models, Walrasian equilibrium are introduced to model the dynamic of the economy $\mathfrak{E}$. This equilibrium is represented by a vector of prices that meets a range of constraints. The second important concept is the notion of Pareto optimality, that defines the allocations at the optimal where no consumer could be made better off without another being made worse off. Note that this statement does not imply egalitarian allocation, but simply the absence of win-win trade-offs in the economic framework (Levin, 2006). Formally, given the productive economy $\mathfrak{E}$:

$$\mathfrak{E} = \left\{ \left(u_j, e_j, \Psi_{i,j}\right), \left(y_i, w_i, r_i\right), i \in \mathfrak{S}, j \in \mathfrak{H} \right\}$$

where each Household $j$, can be defined by its utility $u_j$, endowment $e_j$ and portfolio[26] $\Psi_{i,j}$ representing its positions in firms. On the other side, firms are defined by their production $y_i$, salaries $w_i$ and dividends $r_i$. Under the Pareto optimality condition, the Walrasian equilibrium for production is the vector $\left\{ \left(p_i, x_j, y_i\right), i \in \mathfrak{S}, j \in \mathfrak{H} \right\}$ where $p_i$ is the vector of prices, $x_j$ is the vector translating the allocation of the demand in the population, and $y_i$ reflects the production. The equilibrium exists and, if we focus on production, it satisfies (Levin, 2006):

(i) Firms maximize their profits, $\forall i \in \mathfrak{S}$:

$$y_i^\star = \arg\max\; p_i \cdot y_i - \frac{\mathrm{d}y_i}{\mathrm{d}t} \cdot k_i - \left( y_i - k_i \frac{\mathrm{d}y_i}{\mathrm{d}t} \right) \cdot L_i$$

where $L_i$ is the number of employees.

(ii) Households consume to maximize their utility, $\forall j \in \mathfrak{H}$:

$$\begin{aligned} x_j^\star &= \arg\max u_j\left(x_j\right)\\ \text{s.t.} \quad & p(x_j - e_j) - p \sum_{i \in \mathfrak{S}} \Psi_{i,j} y_i \le 0 \end{aligned}$$

[25] Implies that the demand equals the supply on both the capital and labor markets.

[26] The term portfolio is abusive here, in fact, this parameter refers to the share of the Firm $i$ owned by the Household $j$. The financial market does not formally intervene in the price dynamics for this definition of the Walrasian equilibrium.

(iii) Markets clear[27] (supply matches demand for products):

$$\sum_{j \in \mathfrak{H}} (x_j - e_j) = \sum_{i \in \mathfrak{S}} y_i$$

This definition is one of the possible formalizations of the Walrasian equilibrium for the production economy. There are, however, other ways to write the problem. For instance Levin (2006) defines financial market equilibrium as a collection of portfolios $\Psi^{\star}_{i,j}$, individual consumptions $x_j$, and prices $p^{\star}$ such as agents, here investors, maximize their utility and markets clear. The integration of the environmental dimension in a general equilibrium modeling financial market would require us to compose with these two definitions. If the complexity can be increased to meet this goal, the convergence of this type of model is not straightforward and higher complexity does not always imply better representation of the reality. Therefore, most IAMs use a traditional welfare optimization giving the optimal path for the tax over time. This type of model would be effective if governments and regulators were both willing and able to take action against climate change, which appears not to be the case. Some have indeed advocated that the traditional *Keynesian*[28] approach might not be appropriate to describe climate economy, which is likely to be closely related to private investment and innovation processes. Kaleki's (1939) vision of the economy, opposed to the traditional view of growth with full employment, gives higher importance to investment because it generates capital stock, profits and aggregate demand. The economic module of IMACLIM-R (Cassen *et al.*, 2010)[29] for instance, is justified as follows:

> "*We thus adopted Kaleckian dynamics in which investment decisions are driven by profit maximization under imperfect expectations in non-fully competitive markets (Kaldor, 1957; Kalecki, 1939). Disequilibria are endogenously generated by the inertia in adapting to new economic conditions due to non-flexible characteristics of equipment vintages available at each period. The inertia inhibits an automatic and costless come-back to a steady-state equilibrium*" (Cassen *et al.* 2010, page 7).

The CGE models are, however, limited when it comes to embedding endogenous technical change[30]. In the case of endogenous learning, the linear programing methods used lead generally to non-linear and non-convex optimization problems (Köhler *et al.*, 2006), which make them poor estimators of long-term projections.

All in all, we see that the modeling of production in an infinitely growing economy with a limited natural capital is a conflicting concept. Moreover, the inter-temporal preference is hardly representative of what is likely to occur because of the absence of long-lasting authority. Decentralized general equilibrium models, if they provide better answers in terms of dynamics, are highly sensitive models with potentially high complexity which can end up shifting the problem into black boxing the question into a model with an assumed scientific legitimacy. On the other hand, the seminal model of Nordhaus, the main purpose of which was to give an idea of the optimal tax, was rather simple and comprehensive.

[27] Note that the market clearing condition requires conditions not only on supply and demand for product but also for capital and labor.

[28] See Keynesian economics in the glossary.

[29] This model, developed by the CIRED, makes the interface between static macroeconomic annual equilibrium and bottom-up sub-models representing the evolution of the natural and engineering science modules that affect the economy with regard to sectoral specificity.

[30] See Section 3.2 on page 38.

## 2.2 The seminal model of Nordhaus

The dynamic integrate climate economy (DICE) model is a dynamic optimization model for estimating the optimal path of reductions of greenhouse gas emissions. DICE was the first integrated assessment model with a climate component. It was developed by Nordhaus (1992) and has gone through several extensions and revisions since (Nordhaus and Yang, 1996; Nordhaus and Boyer, 2010; Nordhaus and Sztorc, 2013; Nordhaus, 2017; 2018). It is a highly simplified deterministic model based on a neoclassical view of the economic growth theory. The original model does not distinguish regions or sectors and is based on about 25 equations. Similarly to many IAMs, it was developed in GAMS[31] which stands for general algebraic modeling system. This system was developed by the World Bank in the early 1970s to easily express and solve optimization problems sometimes based on numerous equations.

> "*The aim of this system is to provide one representation of a model which is easily understood by both humans and machines*" (Bisschop and Meeraus, 1982, page 1-2).

This seminal model describes the behavior of the variables with little specification and feedback, but it gives a reliable overview of the problem. The early concept is that the social planner can choose to invest today to preserve consumption in the future. This type of modeling applies to education or technology and was thereby extended to natural capital (Nordhaus and Sztorc, 2013). The main objective of this model is to compute the optimal social cost of carbon $\mathrm{SCC}(t)$ and reduction ratio $\mu(t)$. Therefore, the reduction target maximizing the welfare overtime is an output of this model. In the field of asset management, these targets are used to build aligned portfolios according to varying methodologies that are given in Appendix A.3 on page 70. In this section, we give an overview of how this reduction targets are computed. Moreover, we reiterate that governments use these models to have an idea of the optimal policy, therefore the social cost of carbon obtained is a good proxy of the carbon risks stemming from potential regulations.

### 2.2.1 Model specification

**The neoclassical fundamental model** Following the Solow model, the production output $Y(t)$ is given by the Cobb-Douglas function:

$$Y(t) = A_{\mathrm{TFP}}(t) K(t)^{\alpha} L(t)^{1-\alpha}$$

where $A_{\mathrm{TFP}}(t)$ is the technological progress or total factor of productivity (TFP), $K(t)$ is the capital and $L(t)$ is the labor force proportional to the population. Previously, we did not make the distinction between the production $Y(t)$ and the net output $Q(t)$ because we had the identity $Y(t) = Q(t)$. Nordhaus considers that there is, however, a discrepancy between $Y(t)$ and $Q(t)$ because climate damages generate losses at the global level, implying that:

$$Q(t) < Y(t)$$

[31] The GAMS and Excel versions are available online. GAMS model DICE–2013R (baseline and optimal): http://www.econ.yale.edu/~nordhaus/homepage/homepage/DICE2013R_100413_vanilla.gms.
Excel model DICE–2013: https://github.com/psztorc/DICE/raw/master/models/excel/DICE2013R.xlsm.

More precisely, the net output $Q(t)$ is defined as follows:

$$Q(t) = \Omega_{\text{climate}}(t) \cdot Y(t)$$

where $\Omega_{\text{climate}}(t) < 1$ is the percentage of the output that is lost because of climate change. Nordhaus and Sztorc (2013) assumes that:

$$\Omega_{\text{climate}}(t) = \frac{1 - \Lambda(t)}{1 + D(t)} = \Omega_D \cdot \Omega_\Lambda$$

where $D(t) > 0$ is the climate damage function and $\Lambda(t)$ are the *abatement costs*[32]. The latter, also called mitigation costs, are the cost of reducing GHG emissions. For example, we can assume that $D(t)$ measures the losses implied by natural disasters or production disruption, whereas $\Lambda(t)$ represents for instance the investment costs required to shift from fossil fuel to clean energy sources.

Concerning the economic dynamics, Nordhaus uses the traditional macroeconomic modeling:

$$Q(t) = C(t) + I(t)$$

where $C(t)$ is the consumption and $I(t) = s(t)\,Q(t)$ is the investment[33]. The expression of the consumption becomes:

$$\begin{aligned} C(t) &= (1 - s(t))\,Q(t) \\ &= (1 - s(t))\,\Omega_{\text{climate}}(t)\,A_{\text{TFP}}(t)\,K(t)^{\alpha}\,L(t)^{1-\alpha} \end{aligned} \tag{4}$$

The dynamics of the knowledge factor $A_{\text{TFP}}(t)$, the capital $K(t)$ and the labor force $L(t)$ are defined as follows:

$$\begin{cases} A_{\text{TFP}}(t) = (1 + g_A(t))\,A_{\text{TFP}}(t-1) \\ K(t) = (1 - \delta_K)K(t-1) + I(t) \\ L(t) = (1 + g_L(t))\,L(t-1) \end{cases} \tag{5}$$

where $g_A(t)$ is the growth rate of the technological change, $\delta$ is the rate of depreciation of capital stock and $g_L(t)$ is the time-varying growth of the population. It is assumed that:

$$g_A(t) = \frac{1}{1 + \delta_{g_A}} g_A(t-1)$$

and:

$$g_L(t) = \frac{1}{1 + \delta_{g_L}} g_L(t-1)$$

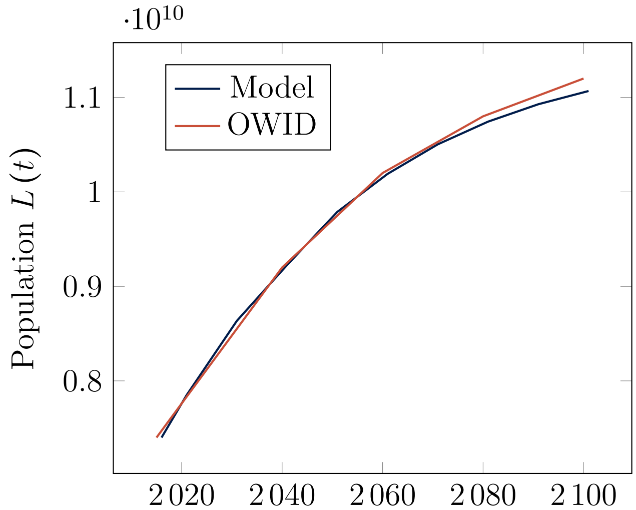
where $\delta_{g_A}$ and $\delta_{g_L}$ are respectively the decline rate of TFP and labor growth. The growth of both population and technical progress are set to evolve exogenously according to the input parameters $g_L(t)$ and $g_A(t)$ and the initial observations for the current stocks and growth rates. Other modeling specifications representative of the asymptotic evolution of these variables are possible

[32] See the glossary for details about abatement costs.

[33] See Section 2.1.1 on page 11. Note that here, we introduce $s(t)$ as a variable of time.

as long as they fit demographic estimations[34]. For instance, Figure 3 compares the labor function used in the DICE–2016R[35] model, with the population estimation given by ur world in data[36] (OWID).

Figure 3: Input trajectory of the population




**Endogenous costs of climate change** We have seen that the output loss ratio due to climate costs is given by:

$$\Omega_{\text{climate}}(t) = \frac{1 - \Lambda(t)}{1 + D(t)}$$

where the climate damages $D(t)$ and the abatement costs $\Lambda(t)$ affecting the production of goods were clearly distinguished but gathered in the expression of the loss coefficient. Climate damages $D(t)$ are represented by Nordhaus with a quadratic function of the atmospheric temperature $\mathcal{T}_{\text{AT}}$ such as:

$$D(t) = a_1 \mathcal{T}_{\text{AT}}(t) + a_2 \mathcal{T}_{\text{AT}}(t)^2$$

where $\mathcal{T}_{AT}(t)$ is the atmospheric temperature $a_1$ and $a_2$ are scale parameters, and $a_1$ is often null. The climate damage coefficient[37] $\Omega_D(t) = (1 + D(t))^{-1}$ represents the fraction of output, most commonly GDP in global macroeconomic models, lost because of the increase in temperatures[38].

[34] In the last version of DICE (Nordhaus, 2018), the dynamics of $L(t)$ is replaced by the following:

$$L(t) = L(t-1) \cdot \left(\frac{L_\infty}{L(t-1)}\right)^{\delta'_{g_L}}$$

[35] See Table 3 on page 31.

[36] https://ourworldindata.org/world-population-growth.

[37] Note that $D(t)$ is often given as a function of the temperature, $D(\mathcal{T}_{\text{AT}})$ in the literature which is unambiguous because of the endogenous evolution of the temperatures with time in these models.

[38] Most consider only damages in the expression of $\Omega_{\text{climate}}$, in other words $\Omega_{\text{climate}} = \Omega_D$, and introduce the mitigation cost elsewhere. Section 3.1 focuses on damage coefficients $\Omega_D$.

Finally, the cost of reduction of greenhouse gas emissions, abatement or mitigation costs, are modeled as follows:

$$\Lambda(t) = b_1 \mu(t)^{b_2}$$

where $\mu(t)$ is the control rate and $b_1$ and $b_2$ scales and nonlinearity parameters. This function is highly simplifying and aggregated and does not, for instance, consider sector specificity (Vogt-Schilb *et al.*, 2013) or potential inertia[39] (Ha-Duong *et al.*, 1997). Combining the cost and damage relationships, we obtain the value of $\Omega_{\text{climate}}(t)$ in the production function:

$$\begin{aligned}
\Omega_{\text{climate}}(t) &= \Omega_D \cdot \Omega_\Lambda \\
&= \frac{1}{1 + D(t)} \cdot (1 - \Lambda(t)) \\
&= \frac{1 - b_1 \mu(t)^{b_2}}{1 + \theta_1 \mathcal{T}_{\text{AT}}(t) + \theta_2 \mathcal{T}_{\text{AT}}(t)^2}
\end{aligned} \tag{6}$$

**Geophysical climate module** This module makes the link between the production of representative goods, the implied increase of GHG concentration in the atmosphere and climate change. We reiterate that in this model, the only variables affecting the output (6) are temperatures and more specifically the atmospheric temperature $\mathcal{T}_{\text{AT}}(t)$, $\mu(t)$ being an endogenous control rate or feedback parameter fixed by an optimization process.

The total emission of GHG $\mathcal{E}(t)$ implied by the production $Y(t)$ at time $t$ follows:

$$\mathcal{E}(t) = (1 - \mu(t))\sigma(t) Y(t) + \mathcal{E}_{\text{Land}}(t) \tag{7}$$

where mitigation policies are translated by the control rate $\mu(t)$, $\mathcal{E}_{\text{Land}}(t)$ represents exogenous land-use emissions, and $\sigma(t)$ is the uncontrolled ratio of GHG emissions to output. In the first version it was assumed to decline exogenously by 1.25% each year and revised in 2016 to −1.5% to fit observations. The variation of this parameter can be integrated into the model with a similar logistic function to labor and productivity:

$$\sigma(t) = (1 + g_\sigma(t))\sigma(t-1)$$

where:

$$g_\sigma(t) = \frac{1}{1 + \delta_\sigma} g_\sigma(t-1)$$

The DICE embeds a reduced form of a global circulation accumulation model describing the evolution of GHG concentration in the atmosphere. The DICE uses a three-reservoir model comprising the atmosphere AT, the upper oceans UP and deep ocean LO that can be considered as an infinite sink for carbon. The set of geothermic layers on which this model is based is consequently $\mathfrak{L}_{\mathcal{C}} = \{\text{AT}, \text{UP}, \text{LO}\}$. The concentrations between layers follow:

$$\begin{cases}
\mathcal{C}_{\text{AT}}(t) = \xi_{1,1}\mathcal{C}_{\text{AT}}(t-1) + \xi_{2,1}\mathcal{C}_{\text{UP}}(t-1) + \mathcal{E}(t) \\
\mathcal{C}_{\text{UP}}(t) = \xi_{1,2}\mathcal{C}_{\text{AT}}(t-1) + \xi_{2,2}\mathcal{C}_{\text{UP}}(t-1) + \xi_{3,2}\mathcal{C}_{\text{LO}}(t-1) \\
\mathcal{C}_{\text{LO}}(t) = \xi_{2,3}\mathcal{C}_{\text{UP}}(t-1) + \xi_{3,3}\mathcal{C}_{\text{LO}}(t-1)
\end{cases}$$

[39] See abatement costs in the glossary for the extended specification.

where $\xi_{i,j}$ represents the flow parameters between reservoirs over the step $\Delta$ considered. Let $\mathcal{C} = (\mathcal{C}_{\mathrm{AT}}, \mathcal{C}_{\mathrm{UP}}, \mathcal{C}_{\mathrm{LO}}) \in \mathbb{R}^3$, the problem becomes:

$$\mathcal{C}(t) = \Phi_{\mathcal{C},\Delta}\mathcal{C}(t-1) + B_{\mathcal{C},\Delta}\mathcal{E}(t) \tag{8}$$

where the matrices $\Phi_{\mathcal{C},\Delta}$ and $B_{\mathcal{C},\Delta}$ are defied as:

$$\Phi_{\mathcal{C},\Delta} = \begin{bmatrix} \xi_{1,1} & \xi_{1,2} & 0 \\ \xi_{2,1} & \xi_{2,2} & \xi_{3,2} \\ 0 & \xi_{3,2} & \xi_{3,3} \end{bmatrix} \quad \text{and} \quad B_{\mathcal{C},\Delta} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

The layers are mostly conservative as shown by the diffusion parameters in Table 1. For instance, more than 79.7% of the $CO_2$ stays in the same layer at $t + \Delta$ after $\Delta = 5$ years. We note that exchanges between layers were under-evaluated in the prior versions. To compare these parameters to other IAMs, we added in the last column of Table 1 the transition matrix used in the WITCH[40] model (Emmerling *et al.*, 2016). We observe that even these physical variables are subject to uncertainty.

Table 1: Concentration diffusion parameters ($\Delta = 5$ years step)

| | DICE–2013 | DICE–2016 | WITCH–2016 |
|---|---:|---:|---:|
| $\xi_{1,1}$ | 91.20% | 88.00% | 88.00% |
| $\xi_{1,2}$ | 3.83% | 19.60% | 12.00% |
| $\xi_{2,1}$ | 8.80% | 12.00% | 4.70% |
| $\xi_{2,2}$ | 95.92% | 79.70% | 94.80% |
| $\xi_{2,3}$ | 0.03% | 0.15% | 0.50% |
| $\xi_{3,2}$ | 0.25% | 0.70% | 0.08% |
| $\xi_{3,3}$ | 99.96% | 99.85% | 99.92% |

Source: Kellett *et al.* (2018), Emmerling *et al.* (2016).

The next step consists in linking accumulated GHG and climate change. General circulation models (GCM) for meteorological forecasting are too complex to be incorporated into economic models. Therefore, Nordhaus focused on average atmospheric temperatures. The relationship between the GHG accumulation and the increase in *radiative forcing*[41] $\mathcal{F}_{\mathrm{RAD}}(t)$ arises from empirical measurements (see for instance Ramanswamy *et al.*, 1991) and climate models[42]:

$$\begin{aligned} \mathcal{F}_{\mathrm{RAD}}(t) &= \eta \ln_2\left(\frac{\mathcal{C}_{\mathrm{AT}}(t)}{\mathcal{C}_{\mathrm{AT}}(1750)}\right) + \mathcal{F}_{\mathrm{EX}}(t) \\ &= \eta \ln_2(\mathcal{C}_{\mathrm{AT}}(t)) - \eta \ln_2(\mathcal{C}_{\mathrm{AT}}(1750)) + \mathcal{F}_{\mathrm{EX}}(t) \end{aligned} \tag{9}$$

[40] Where the carbon-cycle is also a three-layer model calibrated to MAGICC (Meinshausen *et al.*, 2011).

[41] See glossary for radiative forcing.

[42] We reiterate that we have the relationship:

$$\ln_2(x) = \frac{\ln(x)}{\ln(2)}$$

where $\eta$ is the radiative force equilibrium obtained for carbon doubling[43] and $\mathcal{F}_{\mathrm{EX}}(t)$ the exogenous forcing introduced latter (Nordhaus, 2018). The DICE model used Schneider and Thompson's (1981) approach in the most basic way[44]. The introduction of diffusion parameters between layers for both concentrations and temperatures mostly tends to delay reduction actions. In other words, adding complexity to the climate module negatively affects climate action in general. Moreover, one of the parameters embeds carbon absorption by ocean which shifts the problem as it has consequences of marine biota and decrease their pH (Caldeira and Wickett, 2003) and might entail other catastrophic consequences. This model does not reflect the complexity of climate change which would require a regional approach:

> "*The thermal inertia of the upper layers of the oceans, combined with vertical mixing of deeper oceanic waters, could delay the response of the globally averaged surface temperature to an increasing atmospheric CO2 concentration by a decade or so relative to equilibrium calculations [...]. It is found that because of the latitudinal dependence of both thermal inertia and radiative and dynamic energy exchange mechanisms, the approach toward equilibrium of the surface temperature of various regions of the earth will be significantly different from the global average approach*" (Schneider and Thompson, 1981, page 3135).

However, a more local approach would require using advanced GCM for meteorological projections coming with the downscaling issues to estimate climate risks that are specific to each location. In this model, the climate system for temperatures is characterized by a multilayer system comprising the atmosphere and the mixed layer. The simplified temperature module is therefore represented

[43] See in the glossary for more details about equilibrium climate sensitivity (ECS).

[44] Indeed, the radiative forcing defined in the IPCC third assessment report is given for each GHG. On the one hand, the increase of radiative forcing due to $\mathrm{CO}_2$ follows:

$$\mathcal{F}_{\mathrm{RAD}}^{CO_2}(t) = 5.35 \cdot \left(\ln\left(\mathcal{C}_{\mathrm{AT}}^{CO_2}(t)(t)\right) - \ln(592.14)\right)$$

while for other GHG, with interdependent and complex formulation, are approximated differently. For example in WITCH – 2016 (Emmerling *et al.* 2016), we have:

$$\mathcal{F}_{\mathrm{RAD}}^{\mathrm{oghg}}(t) = \mathrm{inter} \cdot \mathrm{fac}\left(\sqrt{\mathrm{stm} \cdot \mathcal{C}_{\mathrm{AT}}^{\mathrm{oghg}}(t)} - \sqrt{\mathrm{stm} \cdot \mathcal{C}_{\mathrm{AT}}^{\mathrm{oghg}}(1750)}\right)$$

where inter, fac, stm are parameters relative to physical properties of the gases for oghg $\in \{CH_4, N_2O\}$. We reiterate that the purpose of this seminal model is not to give a perfectly representative picture of the future, but to have an idea of the optimal reduction.

by a two-layer system[45]:

$$\begin{cases} \mathcal{T}_{\mathrm{AT}}(t) = \mathcal{T}_{\mathrm{AT}}(t-1) + \left(\dfrac{1}{C_{\mathrm{AT}}}\right) \cdot \left(\mathcal{F}_{\mathrm{RAD}}(t) - \lambda \mathcal{T}_{\mathrm{AT}}(t-1) - \gamma\left(\mathcal{T}_{\mathrm{AT}}(t-1) - \mathcal{T}_{\mathrm{LO}}(t-1)\right)\right) \\ \mathcal{T}_{\mathrm{LO}}(t) = \mathcal{T}_{\mathrm{LO}}(t-1) + \left(\dfrac{1}{C_{\mathrm{LO}}}\right) \cdot \gamma\left(\mathcal{T}_{\mathrm{AT}}(t-1) - \mathcal{T}_{\mathrm{LO}}(t-1)\right) \end{cases}$$

where $\mathcal{T}_{\mathrm{AT}}(t)$ and $\mathcal{T}_{\mathrm{LO}}(t)$ are respectively the temperatures of the atmospheric and near surface layers at time $t$, $\mathcal{F}_{\mathrm{RAD}}$ is the radiative forcing relative to the GHG concentration, $C_{\mathrm{AT}}$ and $C_{\mathrm{LO}}$ are the thermal capacity of the two layers, $\gamma$ is the heat exchange coefficient and $\lambda$ is the climate feedback parameter. This equation can be rewritten with $\mathcal{T} = (\mathcal{T}_{\mathrm{AT}}, \mathcal{T}_{\mathrm{LO}}) \in \mathbb{R}^2$ as follows (Kellett *et al.*, 2018):

$$\mathcal{T}(t) = \Phi_{\mathcal{T},\Delta}\mathcal{T}(t-1) + B_{\mathcal{T},\Delta}\mathcal{F}_{\mathrm{RAD}}(t) \tag{10}$$

where:

$$\Phi_{\mathcal{T},\Delta} = \begin{bmatrix} \phi_{1,1} & \phi_{1,2} \\ \phi_{2,1} & \phi_{2,2} \end{bmatrix}, \quad B_{\mathcal{T},\Delta} = \begin{bmatrix} \xi_1 \\ 0 \end{bmatrix}$$

and:

$$\phi_{1,1} = 1 - \frac{\Delta}{C_{\mathrm{AT}}}(\lambda + \gamma), \ \phi_{1,2} = \frac{\gamma\Delta}{C_{\mathrm{AT}}}, \ \phi_{2,1} = \frac{\gamma\Delta}{C_{\mathrm{LO}}}, \ \phi_{2,2} = 1 - \frac{\gamma\Delta}{C_{\mathrm{LO}}}, \ \xi_1 = \frac{\Delta}{C_{\mathrm{AT}}}$$

The matrix format will be preferred especially when adding complexity to the model. By contrast with the concentrations parameters, Table 2 shows that the temperature diffusion parameters of the DICE remained stable between 2013 and 2016.

Table 2: Temperature diffusion parameters ($\Delta = 5$ years step)

| | DICE–2013 | DICE–2016 |
|---|---|---|
| $\phi_{1,1}$ | 86.3% | 87.2% |
| $\phi_{1,2}$ | 8.6% | 8.8% |
| $\phi_{2,1}$ | 2.5% | 2.5% |
| $\phi_{2,2}$ | 97.5% | 97.5% |

Source: Kellett *et al.* (2018).

### 2.2.2 DICE usage

The purpose of this model is to compute the social cost of carbon. The mean to do so is to maximize the welfare over time. The consumption defined page 20 is sensitive to climate risks and

[45] These equations are obtained taking a $\Delta = 1$ step Euler discretization of the following continuous-time dynamics for temperatures:

$$\begin{aligned} C_{\mathrm{AT}}\frac{\mathrm{d}\mathcal{T}_{\mathrm{AT}}(t)}{\mathrm{d}t} &= \mathcal{F}_{\mathrm{RAD}}(t) - \lambda\mathcal{T}_{\mathrm{AT}}(t) - \gamma(\mathcal{T}_{\mathrm{LO}}(t) - \mathcal{T}_{\mathrm{AT}}(t)) \\ C_{\mathrm{LO}}\frac{\mathrm{d}\mathcal{T}_{\mathrm{LO}}(t)}{\mathrm{d}t} &= \gamma(\mathcal{T}_{\mathrm{LO}}(t) - \mathcal{T}_{\mathrm{AT}}(t)) \end{aligned}$$

where $\gamma = C_{\mathrm{LO}}/\tau_2$ is the heat exchange coefficient and $1/\tau_2$ is the transfer rate from upper layer to lower layer.

to the cost of investing to reduce emissions. Therefore, the main feedback parameter will be the control rate, with all other variables being endogenous in this specification. The other variable of interest will be determined along the path to represent the elasticity of consumption with respect to emissions: the *social cost of carbon*[46]. The optimization problem and the notion of social price of carbon will be discussed in this section.

**Optimization problem** In contrast to the production and damages function and the geophysical module that are common to all IAMs, this step concerns only optimization IAMs[47]. In this model, the constant relative risk aversion utility function of the dictatorial social planner is:

$$u(c(t), L(t)) = L(t) \cdot \frac{(c(t))^{1-\theta} - 1}{1-\theta}$$

where $\theta$ is the measure of social valuation of different levels of consumption or rates of inequality aversion (Nordhaus, 1992). Indeed, in the absence of uncertainty in this deterministic model, the household attitude toward risk is only influenced by variation of consumption in time. If $\theta$ tends to 0 there is no risk aversion, and thus no social aversion for inequality. On the other hand, the social welfare becomes egalitarian through generations[48] when $\theta$ tends to 1. In other words, when $\theta$ is small, consumption levels between generations are highly differentiated, and households are willing to accept large variations in consumption over time. The original DICE (1992) model used the utility function at the limit of $\theta$ tends to 1, namely the logarithm or Bernoullian utility function: $u(c(t), L(t)) = L(t)\ln(c(t))$. Let $W$ be the inter-temporal social welfare to maximize as a function of the control rate $\mu(t)$ and the saving rate $s(t)$[49]:

$$W(\mu(t), s(t)) = \sum_{t=1}^{T} \frac{u(c(t), L(t))}{(1+\rho)^t} \tag{11}$$

where the arguments $\{\mu(t), s(t)\}$ represent the optimal control rate and saving rate maximizing the inter-temporal welfare, the other variables being endogenous of the system of equations. The idea is that the social planner has no power over the exogenous population growth, productivity factor or capital, so the only degree of freedom are these two parameters. The optimal pathways are derived by maximizing the social welfare at each step:

$$W^\star = \max_{\mu(t), s(t)} W(\mu(t), s(t))$$

[46]See social cost of carbon in the glossary.

[47]Decentralized frameworks, with sophisticated damage feedbacks or production substitution specifications, are usually evaluation tools, with no optimal control dimension. Indeed, this optimization purpose is to find the optimal trajectory of the control rate maximizing the welfare over time.

[48]We reiterate in Section 2.1.2 on page 14, that the CRRA risk aversion between state and time is the same. Daniel *et al.* (2017) therefore used an Epstein-Zin preference framework to introduce the differences between state and time in a different way. They showed that the optimal carbon price today is high, 100\$/ton$CO_2$, and declines over time, which contradicts most models advocating for slow increasing policies and not to implement too strong and overly costly policies.

[49]Note that the optimal value for the saving rate is often kept constant as shown in Figures 15 to 17 on page 77.

Then, for a set of constraints, fixing for example the maximum temperature or the growth constraint for the mitigation ratio[50], we can define the following optimization problem:

$$\begin{aligned}
\{\mu^\star(t), s^\star(t)\} &= \arg\max \sum_{t=0}^{T} \frac{u(c(t), L(t))}{(1+\rho)^t} \qquad (12) \\
&\text{s.t.} \quad \begin{cases}
A_{\mathrm{TFP}}(t) = (1+g_A(t))\, A_{\mathrm{TFP}}(t-1) \\
K(t) = (1-\delta_K) K(t-1) + I(t) \\
L(t) = (1+g_L(t))\, L(t-1) \\
\mathcal{E}(t) = (1-\mu(t))\sigma(t)\, Y(t) + \mathcal{E}_{\mathrm{Land}}(t) \\
\mathcal{C}(t) = \Phi_{\mathcal{C},\Delta}\mathcal{C}(t-1) + B_{\mathcal{C},\Delta}\mathcal{E}(t) \\
\mathcal{F}_{\mathrm{RAD}}(t) = \eta \log_2(\mathcal{C}_{\mathrm{AT}}(t)) - \eta\log_2(\mathcal{C}_{\mathrm{AT}}(1750)) + \mathcal{F}_{\mathrm{EX}}(t) \\
\mathcal{T}(t) = \Phi_{\mathcal{T},\Delta}(t-1) + B_{\mathcal{T},\Delta}\mathcal{F}_{\mathrm{RAD}}(t) \\
C(t) = (1-s(t))\, \Omega_{\mathrm{climate}}(t)\, A_{\mathrm{TFP}}(t)\, K(t)^{\alpha} L(t)^{1-\alpha}
\end{cases}
\end{aligned}$$

where $\mu^\star(t) \in [0,1]$ and $s^\star(t) \in [0,1]$ and potentially subject to a set of constraints[51].

**The social cost of carbon** This parameter is computed along this path as the optimal price that should be placed on emissions to preserve future generations. This can be intuitively defined as the "*first estimate of the Pigou*[52] *tax that should be placed on carbon dioxide emissions. Indeed, if the SCC is computed along a trajectory in which the marginal costs of emission reduction equal the SCC, the SCC is the Pigou tax*" (Tol, 2008). Therefore, optimizing the control rate $\mu^\star(t)$ is directly reflected in terms of social cost of carbon. A more general definition was proposed by Nordhaus:

> "*This concept represents the economic cost caused by an additional ton of carbon dioxide emissions (or more succinctly carbon) or its equivalent. In a more precise definition, it is the change in the discounted value of the utility of consumption denominated in terms of current consumption per unit of additional emissions. In the language of mathematical programming, the SCC is the shadow price of carbon emissions along a reference path of output, emissions, and climate change*" (Nordhaus, 2011, page 2).

The path for the shadow price is determined using the ratio of Lagrange multipliers of the incremental change in welfare with respect to emissions, $\partial W^\star/\partial\mathcal{E}(t)$, and the incremental change in welfare with respect to the incremental change in consumption, $\partial W^\star/\partial C(t)$. The formula of the SCC is then given by:

$$\begin{aligned}
\mathrm{SCC}(t) &= \frac{\partial W^\star/\partial\mathcal{E}(t)}{\partial W^\star/\partial C(t)} \qquad (13) \\
&= \frac{\partial C(t)}{\partial \mathcal{E}(t)}
\end{aligned}$$

[50] This constraint fixes the feasibility of the tested scenario. For instance, if regulations were forcing a 10% reduction at time $t$, most models will not allow this variable to jump to 90% at the next iteration.

[51] The three first equations describe the exogenous dynamics of the total productivity factor, population and capital (Equation (5) on page 20). The next equation represents the emission of GHG implied by the production of the final ouput (Equation (7) on page 22). After-that there is the description of the GHG concentration dynamics described page (Equation (8) on page 23), the expression of the radiative force (Equation (9) on page 23), the temperatures (Equation (10) on page 25) and finally the consumption (Equation (4) on page 20).

[52] See Pigouvian tax in the glossary.

In most optimization models, the carbon price represented by the $\mathrm{SCC}(t)$ and the optimal control ratio $\mu^\star(t)$ paths are incrementally defined as the output of the optimization process maximizing the welfare[53]. They are indeed the controlling variables emissions to be reduced along the path, which are material for policy makers. The limits of optimization models are their weak empirical applications. Indeed, they are based on strong functional and parametrization assumptions (Köhler *et al*, 2006). On the other hand, evaluation models are recursive partial or general equilibrium models that generate paths for environmental and economic variables without computing their optimal value but with a more robust econometric basis. This empirical approach implies rather backward-looking analysis that can appear inconsistent with forward-looking decision making for energy generation (Köhler *et al*, 2006).

**Applications and results** Figure 4 on page 29 presents the main operating principle of, not only the DICE but almost every IAMs. The right-hand area presents the type of modeling and representative variables that intervene at each step of the process. In simple terms, the production of the global output $Y(t)$ requires the emissions $\mathcal{E}(t)$ which affect the concentration $\mathcal{C}(t)$ and, through the increase of the radiative forcing $\mathcal{F}_{\mathrm{RAD}}(t)$, the temperatures $\mathcal{T}(t)$. This increase of the average temperature reduces the global economic net output $Q(t)$. To preserve consumption over time, the model finds the optimal value for the control rate, $\mu(t)$, decreasing emissions but coming with the costs $\Lambda(t)$. The social cost of carbon is computed along the path, as the ratio of Lagrange multipliers for incremental change in welfare with respect to emission and incremental change in welfare with respect to consumption.

Running the model obviously requires a certain amount of assumptions[54]. These projected scenarios were presented in Nordhaus and Sztorc (2013), compared to other academic models and extended in 2016 and 2017[55]. The results of this model are given for each variable in Appendix A.6 on page 77 in Figures 15 to 17. The scenarios of interest presented by Nordhaus and Sztorc (2013) were the following:

- *Baseline*[56].
  This scenario is the extension of the 2010 active policies (DICE–2013R). It gives a representation of the current evolution of the temperature in the traditional economic framework if no change occurs.

- Optimal.
  This scenario applies the optimal problem (12) with full participation of the nations (regional model aggregated) starting in 2015. This scenario tracks the optimal trade-off between the

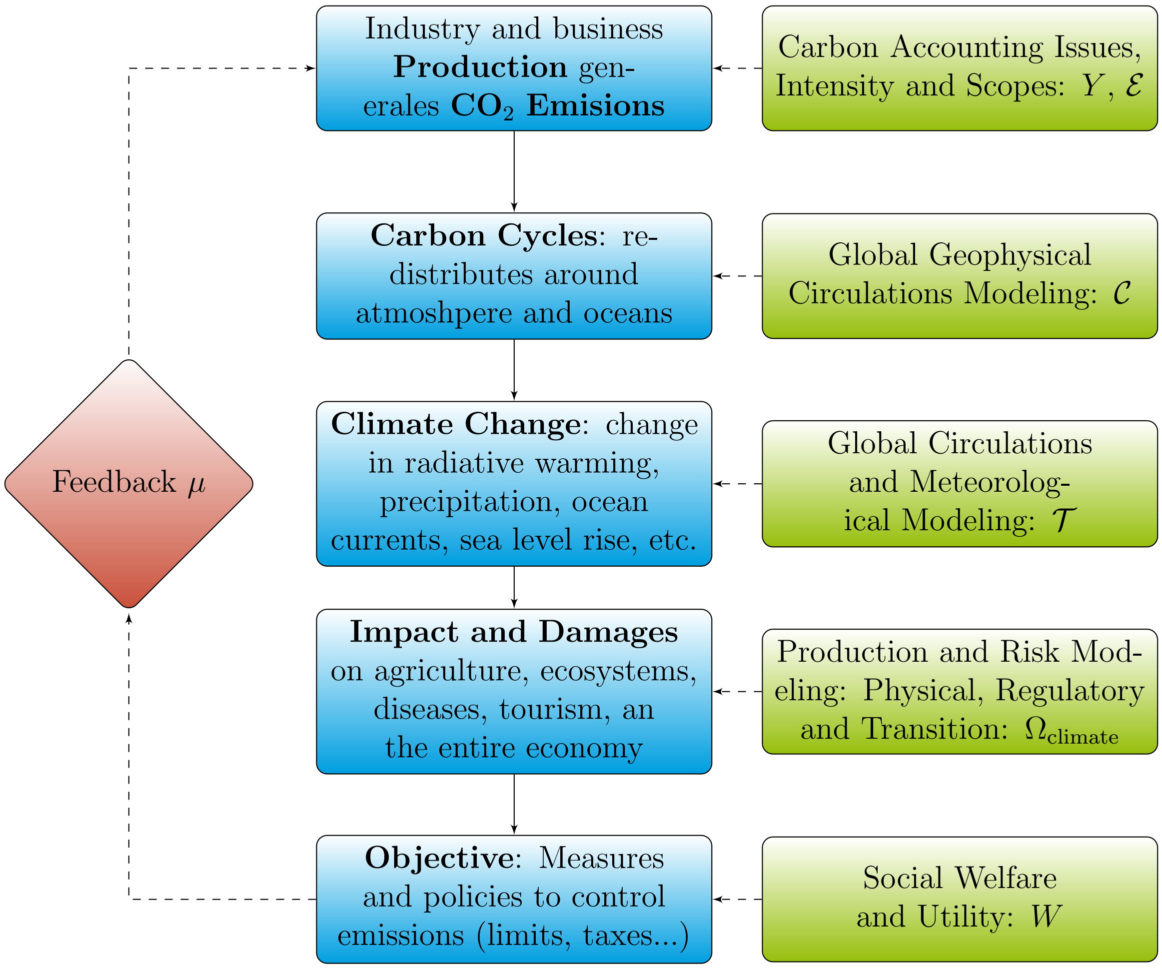
[53]Note that the optimization problem is sometimes posed differently but the fundamental structure is the same. Bansal *et al.* (2016) used a dynamic optimization problem replacing CRRA by and Epstein-Zin recursive utility for example. However, the state vector defined by the set of parameters evolves similarly.

[54]The parameters of the DICE 2013 and 2016 are given in Table 3 on page 31.

[55]Kellett, *et al.* (2018) developed a matlab discrete version of the DICE model, the MPC–DICE available here: `https://github.com/cmkellett/MPC-DICE`. This model's equations are reproduced in Appendix A.5. This model incorporate a receding horizon control process which allows us to better adjust trajectories and can bring an interesting insight if the model were to be extended to the financial system. To replicate infinite horizons simulation we ran this model calibrated to behave as an open-loop infinite-horizon. The program uses an interior point optimizer (IPOPT) to maximize the social welfare under climatic constraints.

[56]See Baseline in the glossary.

Figure 4: Schematic flowchart of the model




Source: Norhaus and Sztorc (2013).

present value of reducing environmental negatives versus maximizing consumption. The results for each variable are given in Figure 16 on page 79 and the parameters used are the ones presented for DICE–2013R version.

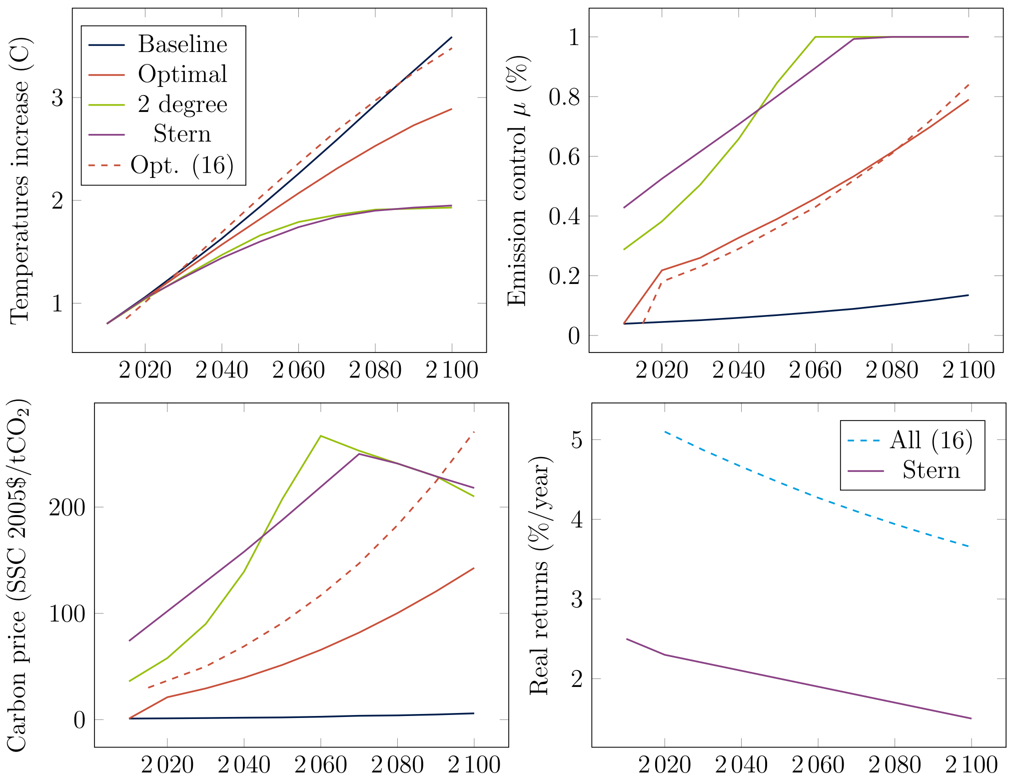
- 2 degree.
  This scenario is obtained by resolving the same optimal problem (12) but with the additional constraint $\max \mathcal{T}_{AT} = 2°\text{C}$. The temperature cannot increase 2°C therefore more effort shall be put into reducing emissions. This problem was solved using DICE–2013R parameters and the output for each variable is given in Figure 15 on page 77.

- Stern.
  This scenario with low discounting generating higher carbon prices computed with DICE–2013R parameters.

- Opt. (16).

Optimal problem (12) solved with 2016 parameters[57]. The results are given in Figure 17 on page 79.

Figure 5: Example of trajectories of the DICE model




Source: DICE model 2013R (Nordhaus and Sztorc, 2013) – 2016R (Nordhaus, 2018).

These runs all lead to their own optimal price and trajectory as shown in Figure 5 on page 30. The 2°C constraint is no longer allowed by the optimizer under the conditions in place since 2016. Moreover, the optimal scenario obtained in 2016 is closer to the baseline than to the most ambitious scenario (Nordhaus, 2018) because of the climate system inertia and the high cost of reducing emissions. The parameter uncertainty[58] (Morgan and Henrion 1990; Edenhofer *et al.*, 2006), characterized by the range of temperatures, and more generally outputs, obtained with little changes in the assumed values of input parameters, is quite a major issue that must be

[57]Note that for optimal (16) carbon prices incorporates Hotelling rents.

[58]Parameter uncertainty denotes that the same model can lead to different outcomes because of uncertain parameters. There is however inconsistency in the literature in terms of terminology. For Edenhofer *et al.* (2006) parameter uncertainty, refers "*to a lack of empirical knowledge to calibrate the parameters of a model to their 'true' values*" and "*there is structural uncertainty or model uncertainty, defined as the uncertainty arising from having more than one plausible model structure*" . Nordhaus (2018) however defined structural as including parameter uncertainty: "*Structural uncertainty, or uncertainty within models, arises from imprecision in knowledge of parameters or variables as well as uncertainty about model structure*" (Nordhaus, 2018). In this paper, we used the definition used by Edenhofer *et al.* (2006) in the Energy Journal dedicated to models comparison.

Table 3: Main parameters of the DICE model

| | | | |
|---|---|---|---|
| **Socioeconomic parameters** | | | |
| | | DICE–2013 | DICE–2016 |
| $\alpha$ | Capital elasticity in production function | 0.3 | 0.3 |
| $\rho$ | Rate of social time preference | 0.015 | 0.015 |
| $\theta$ | Elasticity of shifting consumption | 1.45 | 1.45 |
| $\delta_K$ | Depreciation rate on capital | 0.1 | 0.1 |
| $L_0$ | Initial Population (million people) | 6838 | 7403 |
| $L_a$ | Asymptotic population (million people) | 10500 | 11500 |
| $l_g$ | Growth rate calibrated on 2050 projection | 0.134 | 0.134 |
| $A_O$ | Initial level of total factor productivity | 3.80 | 5.115 |
| $g_a$ | Initial growth rate for TFP per 5 years | 0.079 | 0.76 |
| $\delta_A$ | Decline rate of TFP per 5 years | 0.006 | 0.005 |
| $e_0$ | Initial Industrial emissions | 33.61 | 35.61 |
| $q_0$ | Initial world gross output | 63.69 | 105.5 |
| $k_0$ | Initial capital value | 135 | 223 |
| $\mu_0$ | Initial emissions control rate | 0.039 | 0.03 |
| $\sigma_0$ | Initial CO2-eq-emissions output ratio | 0.5491 | 0.3503 |
| $g_\sigma$ | CO2-eq-emissions output growth scale (1) | 0.01 | 0.0152 |
| $\delta_\sigma$ | CO2-eq-emissions output growth scale (2) | 0.001 | 0.001 |
| **Damage function calibration** | | | |
| $a_2$ | Damages scale coefficient | 0.00267 | 0.00236 |
| $a_3$ | Exponent on damages | 2 | 2 |
| **Mitigation Cost Calibration** | | | |
| $p_n$ | Price backstop technology* (2005US$/tCO2) | 344 | 550 |
| $a_2$ | Exponent of control cost function | 2.8 | 2.6 |
| $\delta_{pb}$ | Initial cost decline backstop cost* | 0.025 | 0.025 |
| **Geophysical variables** | | | |
| $\eta$ | $_{\text{RAD}}$ equilibrium for carbon doubling** | 3.8 | 3.6813 |
| $\xi_1$ $(\frac{\Delta}{C_{AT}})$ | Heat atmosphere capacity over $\Delta$ | 0.098 | 0.10 |
| $\xi_2$ | Equilibrium temperature increase for CO2 doubling** | 12/44 | 12/44 |
| $\mathcal{C}_{\text{AT},1750}$ | Atmospheric concentration in 1750 (GtC) | 588 | 588 |
| $f_0$ | Exogenous $\mathcal{F}_{\text{RAD}}$* (W/m$^2$) | 0.25 | 0.5 |
| $f_1$ | Exogenous $\mathcal{F}_{\text{RAD}}$ threshold* (W/m$^2$) | 0.70 | 1.0 |
| $t_f$ | Time steps until maximum* | 18 | 17 |
| $E_{\text{LO}}$ | Land and oceans absorption coefficient* ($GtCO_2$/year) | 3.3 | 2.6 |
| $\delta_{\text{EL}}$ | Land and oceans absorption variation* ($GtCO_2$/year) | 0.2 | 0.115 |

* The parameters in this table intervene in Kellett *et al.* (2018) MPC–DICE specification described in Appendix A.5 on page 73. Some parameters are not presented in the previous part and are exclusively relative to the algorithm presented in the appendix, however the overall functioning is the same.

** See equilibrium climate sensitivity (ECS) in the glossary.

Source: Kellett *et al.* (2018).

addressed. To analyze this parameter uncertainty Nordhaus (2018) used the symbolic form:

$$Y = H(z, \alpha, u)$$

where $Y$ is the vector of outputs, $z$ a vector of exogenous policies, $\alpha$ a vector of model parameters and $u$ a vector of uncertain parameters. $H$ is the reduced form of the DICE model dynamics. For the set of uncertain parameters, a probability density function was derived and the output can be mapped using a Monte-Carlo sampling of the uncertain variables[59]. The main results for the baseline were a certainty equivalent temperature of 4.10°C (mean = 4.12°C and median = 4.06°C). The social cost of carbon (SCC) in 2015 has a mean 35.6\$/t$CO_2$ with a certainty equivalent of 31.2\$/t$CO_2$. The 2.5°C limit implies that we reach a SCC CE of 229\$/t$CO_2$ in line with Stern's review discounting scenario (266\$/t$CO_2$).

### 2.2.3 A regional version

The regional integrated climate economy (RICE) model is first introduced by Nordhaus and Yang (1996). The weighting process used in this regional version is based on Neigishi theorem (1972) that targets a general market equilibrium on a global weighted environment. It uses Negishi weights (Stanton, 2009), to MERGE (Manne *et al*., 1995) or WITCH (Bosetti *et al*., 2006; 2014). The idea behind this weights was expressed by Elizabeth A. Stanton in an interview:

> "*The importance of making transparent the ethical assumptions used in climate-economics models cannot be overestimated [...]. Negishi weighting is a key ethical assumption at work in climate-economics models, but one that is virtually unknown to most model users. Negishi weights freeze the current distribution of income between world regions; without this constraint, IAMs that maximize global welfare would recommend an equalization of income across regions as part of their policy advice. With Negishi weights in place, these models instead recommend a course of action that would be optimal only in a world in which global income redistribution cannot and will not take place*" (Stanton, 2011).

These weights for each region $r \in \mathfrak{R}$ at time $t$ are given by the relationship:

$$\psi^{(r)}(t) = \frac{\dfrac{1}{u'\left(c^{(r)}(t)\right)}}{\sum_{r \in \mathfrak{R}} \dfrac{1}{u'\left(c^{(r)}(t)\right)}} \tag{14}$$

In the case of isoelastic function (constant elasticity substitution function) the relationship becomes:

$$\psi^{(r)}(t) = \frac{\dfrac{1}{c^{(r)}(t)^{-\theta}}}{\sum_{r \in \mathfrak{R}} \dfrac{1}{c^{(r)}(t)^{-\theta}}} = \frac{c^{(r)}(t)^{\theta}}{\sum_{r \in \mathfrak{R}} c^{(r)}(t)^{\theta}}$$

[59] For computational reasons, it might be preferable to run the model only for some points in the uncertain parameter distribution, for example Nordhaus (2018) runs the DICE for quintiles on the uncertain parameters.

The general preference function is then a Bergson-Samuelson social welfare:

$$W\left(\mu^{(r)}(t), s^{(r)}(t)\right) = \sum_{t} \sum_{r \in \Re} \psi^{(r)}(t) \frac{u\left(c^{(r)}(t), L^{(r)}(t)\right)}{\left(1+\rho^{(r)}\right)^{t}}$$

where $\psi^{(r)}(t)$ is the Neigishi weight of Region $r$ at time $t$ defined by Equation (14) and $\rho^{(r)}$ is the region-specific discount rate. RICE-2010 counted 12 regions – US, EU, Japan, Russia, Eurasia, China, India, Middle East, Sub-Saharan Africa, Latin America, Other high-income countries and Developing countries (Nordhaus and Sztorc, 2013). This decomposition allows varying objectives to be set to some extent according to the welfare of each country. Therefore, it is possible to optimize the aggregated system following, for instance, the Copenhagen Agreement, where richer countries must reduce their emissions more than developing ones[60]. The RICE went through further improvements to fill the gap of the seminal version to embed, for instance, a better representation of the energy sector (Nordhaus and Boyer, 2000). Further models also differentiate sectors within regions as abatement costs can vary between sectors (Vogt-Schilb *et al.*, 2013).

That being said, this change simply affects countries' consumption in a globally aggregated and independent way. The international trade-flows and Armington[61] elasticities of each product are not represented. The major accounting issues posed by GHG reduction targets are exacerbated by the presence of *leakage*[62], spillovers and inconsistent regulatory regimes. Introducing a supranational leakage module would be required to better understand the stakes of the climate impacts. The intermediary consumptions of productive agents are hardly tractable and the levers to reduce negatives are unclear[63]. The modifications of trade roads and the local changing life style due to climate change are ignored in this model. The hypothesis of neutrality of money is also a major issue in every model developed, and the interaction with real variable have not been assessed. This criticism goes back to the trade-off between simplicity and better representativeness of models, which depends on the output we wish to obtain. A possible step further in the field of optimization models would be to develop an open source sectoral and regional dynamic integrated climate economy model, equalizing the utility of sectors to obtain their respective optimal abatement, to help issuers and investors to be aware of their respective trajectory with respect to the environmental stakes.

# 3 Model and scenario comparisons

Nordhaus' scheme, which is illustrated in Figure 4 on page 29, is the backbone of the IAMs but some major updates have been developed by academic practitioners. In other words, if variables $Y(t)$, $K(t)$ $A(t)$, $L(t)$, $\Omega_{\text{climate}}(t)$, $W(t)$, $\mathcal{E}(t)$, $\Lambda(t)$, $\mathcal{T}(t)$ and $\mathcal{C}(t)$, are broadly the same in each model, they can significantly differ in their specifications and the definition of their dynamics.

[60] Reiterating what was noted on page 14, this approach using country-specific utility aims at assisting policy makers and diplomatic discussions to implement 'fair' regulations. One could extend this principle on stakeholders or issuers but the weighting system has to be defined.

[61] This parameter represents the elasticity of substitution between products from different countries. This assumption, according to which the substitution of products traded internationally are differentiated, is generally made in computable general equilibrium models (MIT–ISGM).

[62] See Leakage in the glossary.

[63] See NAMEA, Accounting by issues in the glossary.

Integrated assessment models can be divided in two main classes: policy optimization models and policy evaluation (or simulation) models (Weyant *et al.*, 1996; Nordhaus and Sztorc, 2013), and are broadly used to generate information that is material for policy makers to forecast varying scenarios. The first category of models explicitly provides the optimal feedback affecting the output which can lead to a representative utility maximization. The inputs of these models are parameters and assumptions about the structure of the relationships between variables. The outputs provided by the optimization process are scenarios that depend on a set of constraints. The models that belong to the second category are based on exogenous scenarios and model partial equilibrium between variables. Tol and Fankhauser (1998) and Weyant *et al.* (2006) gather some of the famous examples of IAMs that were used in the energy modeling forum (EMF, 2006). Others have also completed these reviews and compared their results for mitigation or adaptation (Füssel, 2010) or uncertainty of the outcomes of these models (Gillingham *et al.*, 2015). The innovation modeling comparison project (IMCP, Edenhofer *et al.*, 2006) added two types of frameworks to introduce endogenous technical change: the energy system modeling that minimize costs in the energy sector and general equilibrium market balancing demand and supply among agents. There are in fact varying definitions for both economic and climatic models categorization that are confusing for the potential users. Grandjean and Giraud (2017) have proposed a critical review clarifying the underlying concepts and semantics for climate modeling. Indeed, the major difficulty of this subject lies with its multidisciplinary dimension as it requires energy specialists, investors, economists, policy makers and engineers to agree on semantics, accounting and modeling methodologies.

The first source of uncertainty we wish to explore is the specification of damage function $\Omega_{\mathrm{D}}(t)$[64]. Secondly, maintaining production and growth trends in a neoclassical framework usually implies introducing endogenous growth and knowledge as, for example, a form of capital stock affecting productivity[65]. Indeed, endogenous and induced technical change (ETC and ITC) were largely analyzed by academic practitioners. Finally, we will explore further specifications to take interregional trade-flows into account, but also inter-sectoral intermediate demand and network dependencies. Thus, we provide a review of the financial stress-test frameworks providing snapshots of financial and physical exposures accounting for interconnectedness. We conclude this section by a review of the socioeconomic plausible scenarios that allow us to better understand the output of these models.

## 3.1 Climate risks

The computation of the optimal tax[66] is based on the impacts of emission on future economic well-being and thus incorporates damage functions assessing climate risks[67] (see Figure 6). The parameterization of these functions, leading to a climate loss coefficient, is therefore one way

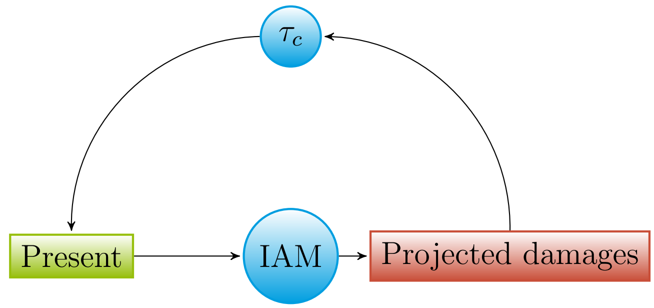
[64]Introduced in Section 2.2.1 on page 21.

[65]See Figure 1 on page 10.

[66]Carbon risk can be defined as agents exposure to the implementation of this tax or pricing system. Similarly the calibration of the carbon pricing system are based on carbon budget itself depending on potential future damages. See the glossary for emission trading system.

[67]See glossary for distinction between carbon, climate and physical risks. This relationship is true for optimization model only. Carbon price is to be interpreted differently according to the modeling structure (see Section 3.4 on page 52).

Figure 6: Relationship between carbon tax $\tau_c$ and damage estimation




to channel climate risks to the financial system[68] despite the huge imperfection of their current definitions. We recall the climate damage coefficient $\Omega_D(t) = (1 + D(\Delta\mathcal{T}_{\text{AT}}(t)))^{-1}$, where $\Delta\mathcal{T}_{\text{AT}}$ represents the increase in atmospheric temperature. This coefficient represents the fraction of output, most commonly GDP in global macroeconomic models, lost because of the increase in temperature. Nordhaus seminal model used a quadratic function of temperature for damages[69]. The numerical damage coefficient implied by this specification is given by:

$$\Omega_D^{\text{N}}(t) = \frac{1}{1 + 2.67 \cdot 10^{-3} \cdot \Delta\mathcal{T}_{\text{AT}}(t)^2} \tag{15}$$

This function is the numerical expression given on page 21 with $a_1 = 0$ and $a_2 = 2.67 \cdot 10^{-3}$. Hanemann (2008) calibrates damages to 7.1% of output at 2.5°C and thus modifies the scale parameter:

$$\Omega_D^{\text{H}}(t) = \frac{1}{1 + 12.07 \cdot 10^{-3} \cdot \Delta\mathcal{T}_{\text{AT}}(t)^2} \tag{16}$$

The shapes of the two specifications are represented as a function of the atmospheric temperatures by the dotted lines in Figure 7 on page 36. Weitzman (2009, 2010, 2012) transformed the damage functions to match the DICE estimate at low temperatures and rise to his suggested values at 6°C and 12°C. To represent his function, we can add onto the two previous functions a higher power of the temperature to the denominator. The resulting functions match respectively Nordhaus and Hanemann functions for low temperatures. For the first we have:

$$\Omega_D^{\text{NW}}(t) = \frac{1}{1 + \left(\frac{\Delta\mathcal{T}(t)}{20}\right)^2 + \left(\frac{\Delta\mathcal{T}(t)}{6}\right)^{6.75}} \tag{17}$$

This function is in red in Figure 7. The second specification with higher damage for low temperature becomes:

$$\Omega_D^{\text{HW}}(t) = \frac{1}{1 + \left(\frac{\Delta\mathcal{T}(t)}{9}\right)^2 + \left(\frac{\Delta\mathcal{T}(t)}{6.5}\right)^{7.5}} \tag{18}$$

[68] Note that Mark Carney identified three transmission channels: the physical risks, the liability risks and the transition risks that are introduced on page 7. Damage functions do not reflect this level of differentiation and simply introduce a global loss coefficient.

[69] See Section 2.2 on page 21.

Equation (18) give the blue logistic shape in Figure 7. Weitzman (2009) indeed suggested to introduce an exponential-quadratic loss function. The theoretical loss function becomes:

$$\Omega_D^W(t) = e^{-\beta(\Delta\mathcal{T}(t))^2}$$

Weitzman's (2009) general definition gives varying results according to the parameter $\beta$. Pindyck (2012) introduced uncertainty using the distribution of the parameter $\gamma$ affecting the growth rate:

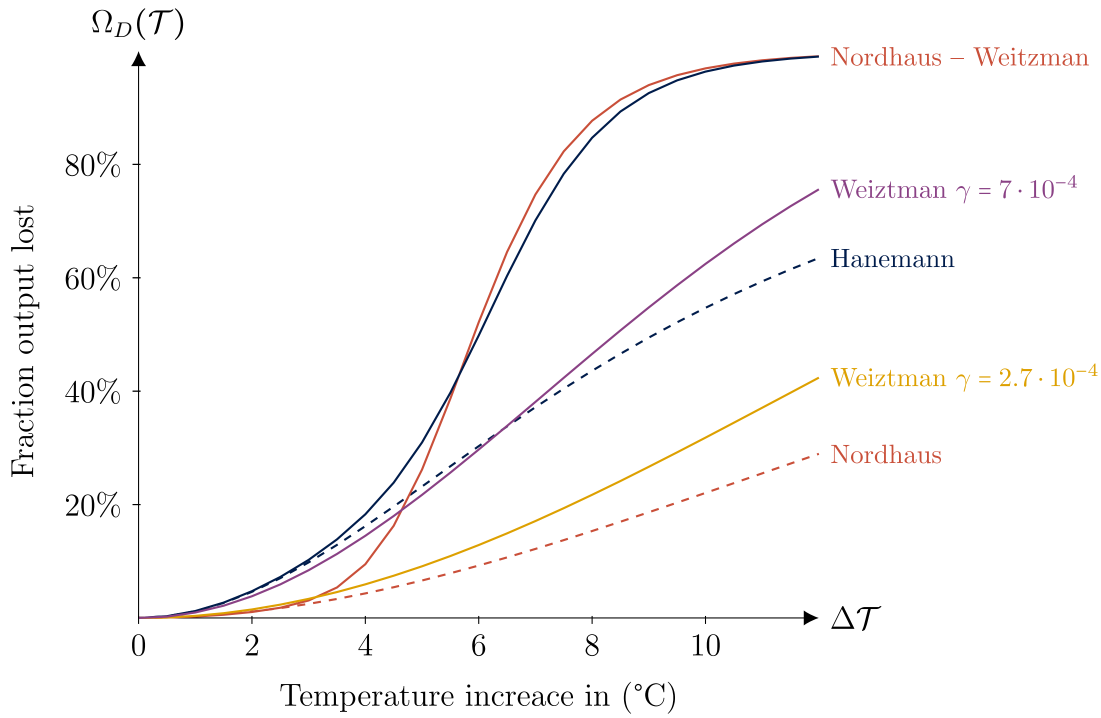
> "*I assume that in the absence of warming, real GDP and consumption would grow at a constant rate $g_0$, but warming will reduce this rate: $g_t = g_0 - \gamma\Delta\mathcal{T}(t)$*" (Pindyck, 2012, page 7).

This parameter is included in the model of Daniel *et al.*, (2018), as follows:

$$\Omega_D^{\mathrm{DLW}}(t) = e^{-13.97\gamma(\Delta\mathcal{T}(t))^2}$$

where $\gamma$ can be drawn from a displaced gamma distribution (Daniel *et al.*, 2018, page 18). We plot in Figure 7 some examples of the possible damage functions than can be obtained with this modeling as a function of $\gamma$. We notice that when $\gamma$ is high, we obtain functions with logistic shapes.

Figure 7: Possible forms of the damage function




Most function used in the literature were quadratic or logistic. Figure 7 shows these two profiles. To obtain similar profiles, the IAM RESPONSE (Dumas *et al.*, 2013) introduced the following damage function:

$$D(t) = a_\phi (\Delta\mathcal{T}(t))^{1+\phi} + \frac{d}{1 + e^{(\Delta\mathcal{T}^\star - \Delta\mathcal{T}(t))/\eta_2}}$$

where $a_\phi$ and $\phi$ are respectively the scale parameter and the exponent parameter for polynomial terms, $d$ and $\eta_2$ represent the influence of logistic effect over a *tipping point*[70] or threshold temperature. For example, $\phi = 1$, $d = 0$ is the quadratic case, and the sigmoid case ($\phi = 0$). It introduces a threshold temperature increase $\Delta\mathcal{T}^\star$ above which the jump is triggered in the sigmoid case.

The definition of this function is at the heart of the evaluation of climate risk exposure. However, damages are mostly globally defined, with little malleability when it comes to distinguishing damages occurring at specific location or impacting specific stakeholders. These deterministic functions hardly apply to financial assets and do not differentiate risks of stranding, liability or transition. In macroeconomic modeling, average temperature is considered to be a good proxy for the global state but this modeling is not scalable to corporate-specific business. To properly express climate risks exposure, a family of damages functions, scaled on each impacting parameter[71], needs to be introduced. The calibration of these functions based on non-existent data (out of sample) is a great challenge for modelers. More importantly, most parameters are fixed arbitrarily to fit with a given estimation at a certain temperature. These functions are therefore subject to much criticisms because of their form and unverifiable parametrization. For instance, Figure 7 shows that we cannot truly distinguish which coefficient will be effective before we passed 2°C. Another criticism is that the deterministic damage functions used in most IAMs have been shown to underestimate optimal tax:

> "*The uncertainty associated with anthropogenic climate change implies carbon taxes that are much higher than implied by deterministic models. This analysis indicates that the absence of uncertainty in DICE2007 and similar models may result in substantial understatement of the potential benefits of policies to reduce GHG emissions*" (Cai *et al.*, 2013, page 2).

To focus now on the global market and the effect of climate on prices, few attempts to project climate risks on the financial system were published. Bansal *et al.* (2016) proposed a temperature augmented long-run risk (LRR-T) model, to account for climate impacts on equity valuation and consumption:

> "*Quantitatively, in the data, a one degree Celsius increase in temperature leads to about* $-5\%$ *decline in equity valuations. [...] In particular, in our baseline LRR-T model, a one degree Celsius increase in temperature lowers the price of the consumption claim by about* $1.74\%$" (Bansal *et al.*, 2016, page 19).

However, this paper also used a quadratic damage function to estimate losses which expose these results to the previous criticism. The main purpose of this exercise, which consists in calibrating future damages, is to compute the optimal price of carbon today and generally advocate for immediate action[72]. Hence, arguing about the form of this function might be, economically speaking,

[70]See tipping point in the glossary.

[71]The FUND model has a more detailed specification of damages. The source code, data, and a technical description of the model are public `www.fund-model.org`.

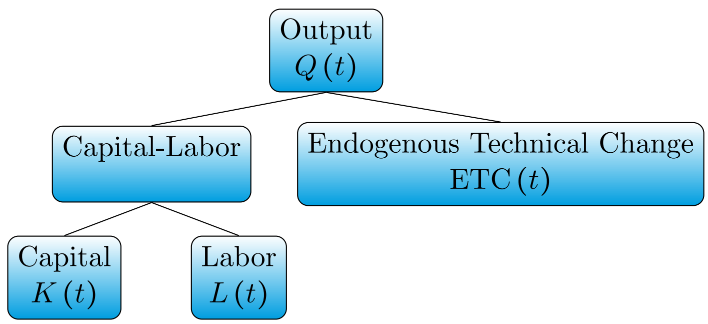
[72]Which had little influence on effective carbon prices worldwide. Therefore it makes sense to reproduce this type of studies, based on highly hypothetic damage functions calibrated to encourage actions, at a portfolio level to extend the potential audience to investors and financial actors. However, quantitatively speaking, the legitimacy of the results will be questionable as long as these functions are involved in the process.

a misguided subject, while focusing on the possible transition pathways leading to zero industrial emissions is certainly an interesting topic that requires further attention.

## 3.2 Modeling technical change

The flourishing literature on climate integrated economy provides a large range of modeling propositions to identify the levers for transforming the economy to reduce environmental negatives. The definition of economic output, specified as a simple function of capital, labor and productivity factor, lacks transparency in terms of energy use and of GHG emissions. This section focuses on the definition of the net production $Q(t)$ to study how modelers have represented the energy and knowledge modules and particularly the endogenous or induced technical change.

Figure 8: Traditional capital–labor vs. endogenous technical change




The structure of the models follows the scheme in Figure 8. The inputs integrate an energetic module allowing endogenous progress. Formally, this figure can be translated in the following constant elasticity substitution relation:

$$Q(t) = \Omega_{\text{climate}}(t)\left(\underbrace{A_{\text{TFP}}(t)\left(K(t)^{\alpha} L(t)^{1-\alpha}\right)^{\vartheta}}_{\text{Traditional Solow production}} + \underbrace{A_{\text{EN}}(t)\,\mathrm{ETC}^{\vartheta}(t)}_{\text{Energy stock evolving endogeneously}}\right)^{1/\vartheta} \tag{19}$$

where $\mathrm{ETC}(t)$ is the function of interest of this section, $\vartheta$ represents the elasticity of substitution between the capital–labor and the energy composite and $A_{\text{EN}}$ is the energy technological progress[73]. There have being several attempts to specify the behavior of this form of nested production in the literature. We therefore propose a non-exhaustive review of the mathematical definitions of the induced technical change, which can be defined as the growth spurt in innovation implied by climate change. Concretely, the inducing phenomena for these changes can be regulatory policies and taxation (optimization models) or the '*demand-pull*' (Ruttan, 2010) for cleaner production or climatic incident and shocks (partial and general equilibrium models).

**Learning-by-doing** Van der Zwaan *et al.* (2002) developed the de-carbonization model with endogenous technologies for emission reductions (DEMETER) to introduce endogenous technical change:

[73]Sometimes calibrated on an '*autonomous energy efficency increase*' (AEEI) factor.

> "*Technological improvements no longer fall as 'manna from heaven', but depend on up-front investments*" (Van der Zwann *et al.*, 2002, page 2).

It is a top-down growth optimization model where the production side is able to choose between cleaning its future production or keep using fossil energy input including the maintenance and operation efforts for both sources[74]. The energy demand and technological change can be endogenized according to the scenario. Concretely, induced technological change lowers the cost of non-fossil input via a learning-by-doing process[75]. To assess the effect of this phenomenon, the authors described the production with the single constant elasticity substitution (CES) function. This paper used Equation (19) but specifically focused on endogenous technical change, therefore, there were no specifications for damages $\Omega_{\text{climate}}(t) = 1$ and:

$$\text{ETC}(t) = \left(\text{EN}_{\text{FOS}}(t)^{\varphi} + \text{EN}_{\text{RNW}}(t)^{\varphi}\right)^{1/\varphi}$$

where $\text{EN}_{\text{FOS}}(t)$ is the fossil energy input and $\text{EN}_{\text{RNW}}(t)$ is the non-fossil energy input. The parameter $\varphi$ represents the elasticity of substitution between fossil and non-fossil energy use. They also introduced the following distribution of the output:

$$Q(t) = C(t) + I_{\text{C}}(t) + I_{\text{FOS}}(t) + I_{\text{RNW}}(t) + M_{\text{FOS}}(t) + M_{\text{RNW}}(t)$$

where $I_{\text{C}}(t)$ is the output saved for future consumption, $I_{\text{FOS}}(t)$ and $I_{\text{RNW}}(t)$ are respectively the investments made in fossil and non-fossil energy inputs and $M_{\text{FOS}}(t)$ and $M_{\text{RNW}}(t)$ are the respective maintenance and operational (M&O) costs. The learning-by-doing phenomenon, that will eventually act on both the required investments and M&O costs of renewable energies, is described thanks to a cumulative capacity:

$$X_{\text{EN}_{\text{RNW}}}(t) = X_{\text{EN}_{\text{RNW}}}(t-1) + \underbrace{\text{EN}_{\text{RNW}}(t-1) - (1-\delta)\,\text{EN}_{\text{RNW}}(t-2)}_{\text{Contribution of the stock over } \Delta t = 1}$$

where the right term of the equation in the contribution over the period considered[76]. The scaling functions $h(X)$ are the key element to introduce the learning-by-doing phenomenon[77] as they allow the capital requirement for both investment and operational management to be defined:

$$I_{\text{RNW}}(t) = a_{\text{RNW}} \cdot h\left(X_{\text{EN}_{\text{RNW}}}(t)\right) \cdot \Delta X_{\text{EN}_{\text{RNW}}}(t)$$

and:

$$M_{\text{RNW}}(t) = b_{\text{RNW}} \cdot h\left(X_{\text{EN}_{\text{RNW}}}(t)\right) \cdot \Delta X_{\text{EN}_{\text{RNW}}}(t)$$

The calibration of the learning curves become the central issue of this type of modeling.

---

[74] They investigated four scenarios in addition to the baseline. Note that this one lead to an 2.4°C increase in temperatures by 2100, which is significantly lower that the one obtained with the unconstrained DICE in the previous section. A model with carbon capture and storage, DEMETER-CCS, was described in a further publication (Gerlagh, 2006).

[75] "*The investment costs of specific technologies are – via so-called learning curves – explicitly linked to the cumulative installed capacity. This reflects the notion of learning-by-doing: the costs of specific energy technologies decrease as commercial investments and installed capacities accumulate*" (van der Zwann *et al.*, 2002, page 2).

[76] The step commonly used in the literature is five years.

[77] "*This scaling function expresses that with little cumulative capacity installed, it takes relatively more energy-specific capital and M&O efforts to produce a given level of energy than when a high level of cumulative capacity is available*" (van der Zwann *et al.*, 2002, page 7).

**Learning-by-researching** The consideration of learning-by-researching corresponds to the introduction of R&D spending in addition to the accumulation of production capacities. For instance, The *model for evaluating regional and global effects* (MERGE), developed by Manne and Richels (1995), is an IAM developed to reflect these effects[78]. MERGE follows an initiative to introduce endogenous learning via R&D spending for carbon-free energy technologies. In particular, some technologies for example, gas-fuel cell with removal, appear in 2020 if sufficient dedicated R&D investment is made. A more sophisticated version was published in 2005 to define the endogenous learning formulation.

Popp (2004) introduced a modified version of the aggregated DICE (ENTICE-BR) to incorporate endogenous technical change. This model considers that energy prices decrease with the learning curve but in comparison to DEMETER, it includes both a backstop technology and technological progress through R&D. The gross production $Q(t)$ is not directly derived from Equation (19) and is defined as follows:

$$Q(t) = \Omega_D A_{\mathrm{TFP}}(t) K(t)^{\alpha} L(t)^{1-\alpha-\vartheta} \mathrm{ES}(t)^{\vartheta} - p_{\mathrm{FOS}}(t) \mathrm{EN}_{\mathrm{FOS}}(t) - p_B(t) \mathrm{B}(t)$$

where $\mathrm{ES}(t)$ are energy services, $\mathrm{EN}_{\mathrm{FOS}}(t)$ represents the fossil fuels and $\mathrm{B}(t)$ the backstop carbon-free technologies with their respective prices $p_{\mathrm{FOS}}(t)$ and $p_{\mathrm{B}}(t)$. This form of production function was introduced in the formulation of RICE–99 (Nordhaus and Boyer, 2000) but without the distinction between fossil and non-fossil sources nor the introduction of a backstop technology[79]. The ENTICE-BR model introduces a two-nest CES to describe the behavior of energy related component $\mathrm{ES}(t)$[80]:

$$\mathrm{ES}(t) = \left(\alpha_{\mathrm{EN}} \mathcal{K}_{A_{\mathrm{EN}}}(t)^{\rho_A} + \left(\left(\frac{\mathrm{EN}_{\mathrm{FOS}}(t)}{\alpha_{\mathrm{FOS}} \mathrm{EC}(t)}\right)^{\rho_{\mathrm{B}}} + \mathrm{B}(t)^{\rho_{\mathrm{B}}}\right)^{\rho_{\mathrm{B}}/\rho_A}\right)^{1/\rho_A}$$

where $\alpha_{\mathrm{EN}}$ and $\alpha_{\mathrm{FOS}}$ are scaling factors and $\mathrm{EC}(t)$ represents the remaining exogenous change in the ratio of carbon emissions per unit of service[81]. The deepest nest represents the substitution of a backstop over fossil fuels introduced by van der Zwaan *et al.* (2002). The other represents the allocation in R&D dedicated to climate change. The traditional allocation of the output between consumption, investment, research efficiency and research for backstops are following the relationship:

$$Q(t) = C(t) + I_C(t) + I_{A_{\mathrm{EN}}}(t) + I_{\mathrm{B}}(t)$$

[78]Note that the ETC module follows:

$$\mathrm{ETC}(t) = \mathrm{EN}_{\mathrm{EL}}(t)^{\varphi} \cdot \mathrm{EN}_{\mathrm{NE}}(t)^{1-\varphi}$$

where $\mathrm{EN}_{\mathrm{EL}}$ is the electric energy-related value added and $\mathrm{EN}_{\mathrm{NE}}$ is the non-electric input. The form differs slightly from the one previously introduced but the substitution properties are similar.

[79]"*The current RICE-99 and DICE-99 models do not include backstop technologies. Omitting a backstop technology implies that the price of carbon energy can rise to extremely high levels in the future. [...] Experiments indicate that the effect of adding a backstop technology is relatively small over the next century and not worth the additional complexity*" (Nordhaus and Boyer, 2000, page 20).

[80]"*The total energy requirements for production must be met either by the use of fossil fuel or by technological advances that substitute for fossil fuels*" (Popp, 2004, page 7).

[81]Calibrated to fit the DICE model without R&D.

where $I_{A_{\mathrm{EN}}}(t)$ and $I_{\mathrm{B}}(t)$ are respectively the investment dedicated to improving energy efficiency or to backstop technology. The knowledge $\mathcal{K}$ is cumulative through time for both efficiency $A_{\mathrm{EN}}(t)$:

$$\mathcal{K}_{A_{\mathrm{EN}}}(t) = h\left(I_{A_{\mathrm{EN}}}\right) + \left(1 - \delta_{A_{\mathrm{EN}}}\right)\mathcal{K}_{A_{\mathrm{EN}}}(t-1)$$

and for the backstop $B(t)$:

$$\mathcal{K}_{\mathrm{B}}(t) = h\left(I_{\mathrm{B}}\right) + \left(1 - \delta_{\mathrm{B}}\right)\mathcal{K}_{\mathrm{B}}(t-1)$$

where $h$ is here a function describing the learning process, called the '*innovation possibility frontier*' (Popp, 2004)[82]. The price of the backstop is defined similarly as for experience curves and use the progress ratio:

$$p_{\mathrm{B}}(t) = p_{\mathrm{B}}(0)\,\mathcal{K}_{\mathrm{B}}(t)^{-\eta_3}$$

where $\eta_3$ is the parameter representing effect of backstop energy knowledge on prices. This model also embeds an interesting crowd out dimension which appears to be significant in this model[83]:

> "*In the base case, with partial (50%) crowding out of other R&D from new energy R&D, ITC improves welfare by 9.4%. This falls as low as 1.9% with full crowding out, and increases to as much as 45.3% without crowding out*" (Popp, 2004, page 4).

This phenomenon shows that climate transition can only be achieved through private investment. The heterogeneity of investors' beliefs and their willingness to transit to a greener economy is the remaining uncertain parameter for evaluating the future trajectory in a Kaleckian model.

**Hybrid energy models** The two previous paragraphs were dedicated to the modeling standard for global transition in a quite limited environment of possibilities[84]. The missing piece is to link this global approach to bottom-up decision making to select energy sources. The *world induced technical changed hybrid* (WITCH) model, similarly tracks R&D investments with the introduction of the energy services $\mathrm{ES}(t)$. It was first introduced by (Bosetti *et al.*, 2006) and is designed to fill this gap between economic top-down models and bottom-up solutions focusing on the technological dimension. The optimal investment strategy includes a detailed representation of the energy supply in game theory framework presented in Figure 9. The model is based on the same scheme presented in Equation (19) and gives the net output for the region $r \in \mathfrak{R}$:

$$Q^{(r)}(t) = \Omega^{(r)}_{\mathrm{climate}}(t)\,A_{\mathrm{TFP}}(t)\left(a_r\left(K^{(r)}(t)^{\alpha}\,L^{(r)}(t)^{1-\alpha}\right)^{\vartheta} + \left(1 - a_r\right)\mathrm{ES}^{(r)}(t)^{\vartheta}\right)^{1/\vartheta}$$

[82] In this paper, Popp (2004) also discussed the distinction between learning-by-doing vs. learning-by-researching based on empirical evidence and references to prior works.

[83] Crowding out refers to government spending failing to increase overall aggregate demand. In the specific case of climate change, as demonstrated by Popp (2004), higher government spending in new energy causes an equivalent (or 50%) fall in private sector spending and investment. The question of crowd out is anecdotal in IAMs literature, but not for investors. Indeed, expecting government or central banks to take action to face the climate crisis, similarly to what they did for the financial crisis, investors stay on their position because they do not expect instantaneous premia investing on R&D for disruptive energy sources. Despite the lack of strong action from government the mechanism behind the crowding out phenomenon is, so to speak, already on.

[84] The two dimensions were either fossil vs. non-fossil or electric vs. non-electric. Note however that MERGE introduces technological possibilities that were not detailed here.

where the energy service $\mathrm{ES}(t)$ also follows a similar definition:

$$\mathrm{ES}^{(r)}(t) = \left(\alpha_{\mathrm{ES}}\mathcal{K}^{(r)}(t)^{\vartheta_2} + (1-\alpha_{\mathrm{ES}})\left(\mathrm{EN}^{(r)}(t)\right)^{\vartheta_2}\right)^{1/\vartheta_2}$$

where $\alpha_{\mathrm{ES}}$ a substitution parameter between knowledge accumulation and energy spending, $\mathcal{K}^{(r)}(t)$ is the energy knowledge that follows the relationship of accumulation[85]. The particularity of this modelling structure is to embed several substitutions elasticities. The first for the integration of the energy module in the production function (over the traditional aggregate capital–labor[86], see Figure 9) and the others within the definition of the energy module. This way a nested CES function is introduced where substitution parameters between sources are given. The ease of substitution is represented by $\rho$, the case $\rho = 1$ being the linear or perfect substitutes function[87]. For instance, for WITCH the electric sourcing '*Electric (2)*' $E_2(t)$, allows us to substitute fossil fuel generation, nuclear or wind and solar energy $\rho_{E_2} = 2$[88].

The motion of capital becomes:

$$K^{(r)}(t+1) = K^{(r)}(t)(1-\delta_K) + I_C^{(r)}(t) - I_{\mathrm{R\&D}}^{(r)}(t)$$

where $I_{\mathrm{R\&D}}^{(r)}(t)$ is the investment dedicated to transition. This model has been extended and the version described in Emmerling *et al.* (2016) paper is far more advanced and enables complex scenarios[89] to be projected.

This section has shows that there are different ways to model induced technical change in a forward-looking environment. The differences between models are based on their parameters, functional descriptions and respective complexities. Models involving backstops are criticized because they rely on non-existent technologies. On the other hand, they also advocate for increasing private[90] spending on either research for backstop or incremental energy efficiency improvements. If the optimal taxes, given by macroeconomic optimization models, are not directly relevant because they cannot be properly implemented, the optimal R&D investment dedicated to the transition can be assessed. The implementation of '*optimally controlled investment strategies*' based on an optimal amount invested in the transition is, quantitatively speaking feasible. The main pitfalls for implementation currently concern data availability for the detailed breakdown of capital expenditure and abatement cost and requires the continuous reassessment of learning curves or technology prices at a microeconomic level.

[85] The relationship of accumulation for knowledge in energy as a similar form in most IAMs:

$$\mathcal{K}_{\mathrm{EN}}(t) = h(I_{\mathrm{EN}}) + (1-\delta_{\mathrm{EN}})\mathcal{K}_{\mathrm{EN}}(t-1)$$

[86] The standard elasticity of $\rho_{\mathrm{EVA}}$ for energy value added is 40%-50% (Bosseti *et al.*, 2014).

[87] Similarly for inter-temporal substitutions, the elasticity of substitution is $\dfrac{1}{1-\rho}$.

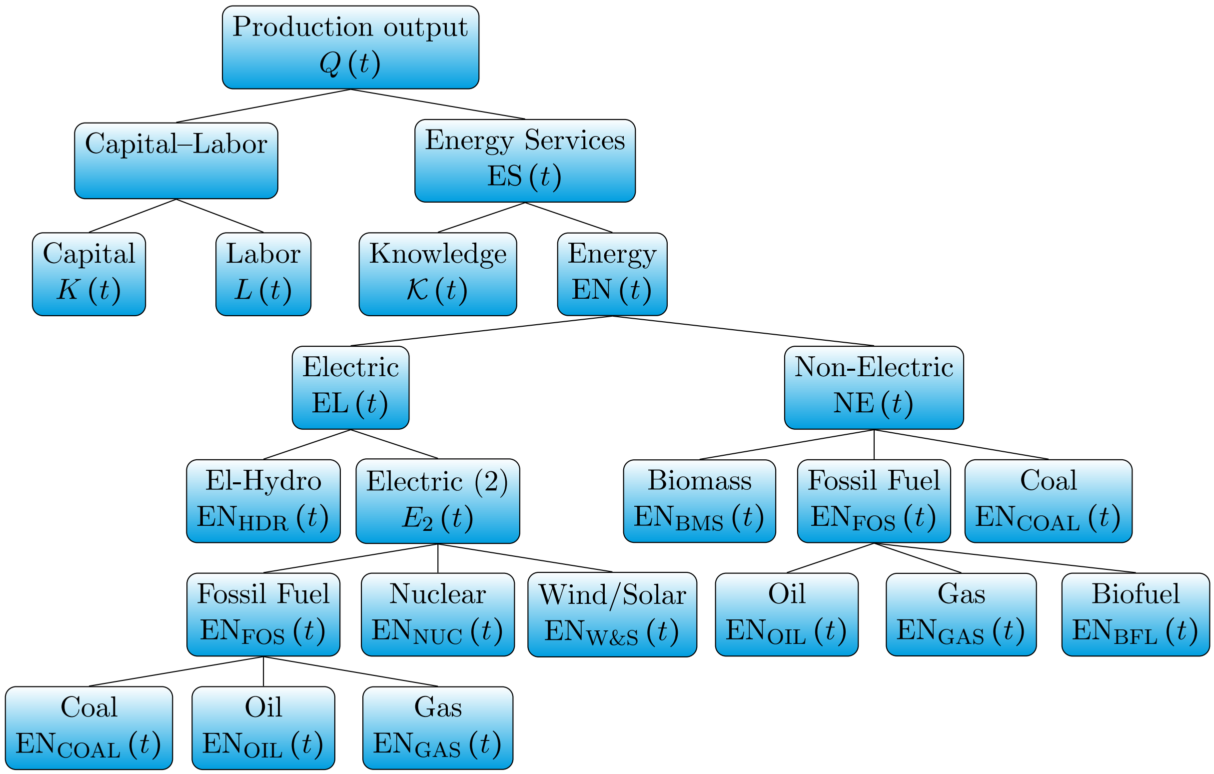
[88] Witch gives for this node:

$$E_2(t) = \sqrt{\mathrm{EN}_{\mathrm{FOS}}(t)^2 + \mathrm{EN}_{\mathrm{NUC}}(t)^2 + \mathrm{EN}_{\mathrm{W\&S}}(t)^2}$$

[89] In particular, this paper presents the adaptation of this modeling scheme to the shared socioeconomic pathways presented in Section 3.5 on page 53.

[90] Because public spending implies a crowd out.

Figure 9: WITCH model production nest structure




Source: Bosetti *et al.* (2014).

## 3.3 Interconnected Models

Most aggregated approaches of climate change ignore the importance of system interconnectedness, cascading effect and international trade-flow. This substantial interconnectivity can possibly be at the root of a new financial crisis and is the source of concern for financial institutions. To some extent, the liability risks, arising from parties suffering losses and seeking for payment from those they judge responsible, are physical risks channeled by the legislative or financial system. This transmission of climate risks among actors demonstrates that the evaluation of the economic climate risks requires a modular approach, with different specifications for each impacting actors. The academic literature has tackled this aspect of prime importance to understand the effect of climate change.

### 3.3.1 International and CGE models

To approach the macroeconomic dimension of climate change one can use either an international IAM including trade-flows or a computable general equilibrium focusing on the efficiency of market responses[91]. In the first, the challenge is to find the level of aggregation that best reflects the

[91] Ultimately general equilibrium models obviously embed international trade-flows but we decide to distinguish here the '*international and input-output-like*' models from the general equilibrium convergence models.

complexity of endogenous macroeconomic dynamics in order to provide information on long-term dynamics. The latter are designed to reproduce dynamics of the real market. They can provide relatively short- and medium-term price estimates and possibly embed a stochastic dimension. A macroeconomic statistical assessment module calibrated on past and real-time data would bring complementary information about the effective market behavior and assist short-term decisions. In this section, we introduce some academic initiatives to approach the concept of system dynamics and the application of computable equilibrium models to climate change.

Global dynamic models give a representation of every variables of interest in a dynamic integrated economy. The DICE is a highly simplified version that aims to define the optimal tax. To track more subtle change in trends or international markets reactions to political and environmental changes, models must embed to some extent further specifications. As an example, AIM/Global-Dynamics optimization model was developed by Masui *et al.* (2006) to estimate the effects of energy-saving technologies and energy efficiency. The entries are deterministic and it follows the RICE structure for the aggregation of regional utilities. This model considers inter-regional trade-flows, is structured using twelve equations that represent each region interacting on the international market. To some extent, this type of model could rely on an input-output matrix to fit the current flows. The schematic structure of this type of model is presented in Figure 10 on page 47, which give a better understanding of the global system dynamic. This type of modeling presents a first representation of the macroeconomic complexity but remains highly schematic and simplified. The production function is a nested CES for non-energy sector, fossil fuel producers and electric producers following a similar structure as Figure 9. The strength of this choice of modeling lies on, for instance, the potential estimation of the cascading effects of trading agreements, if the model is properly calibrated. Moreover, this structure allows some transition strategies to be applied, for instance testing over-investing on some key sectors in key regions[92].

Golosov *et al.* (2014) developed a DSGE focused on energy sourcing for production[93]. This model provides an extension of the previous ones focusing on the interaction between the interaction between energy and non-energy sectors. The market is specified with a two-sided – energy producers and firms consuming for production – model in which each representative agent maximize its profits. The energy $\mathrm{EN}(t)$ is the output of the producing side while it is an additional

[92]One of the main challenges for managers dealing with climate change is not to blindly focus on portfolio decarbonization methodologies based on questionable data, but to consider the transition pathways with further attention. Optimal allocation might require increasing emissions now in some sectors, to reach the global target. For instance, the extraction and transportation of raw material $M(t)$ to build, renewable energy networks is likely to emit GHG, but will most likely improve the global state. The consideration of the entire network is therefore required to finance the optimal path.

[93]They defined the production of the final output with a traditional Solow model specified as follows:

$$Y(t) = \mathrm{EN}(t)^{\nu} \cdot A(t) K(t)^{\alpha} L(t)^{1-\alpha-\nu}$$

where $\mathrm{EN}_0(t) = (\mathrm{EN}_{0,1}(t) .., \mathrm{EN}_{0,n_{\mathrm{EN}}}(t))$ "*denoting a vector of energy inputs used in this final sector at t*" (Golosov *et al.*, 2014). In the production function, which they specified including the damage function, $\mathrm{EN}(t)$ is an energy composite such as:

$$\mathrm{EN}(t) = \left(\kappa_1 \mathrm{EN}_1(t)^{\vartheta_2} + \kappa_2 \mathrm{EN}_2(t)^{\vartheta_2} + \kappa_3 \mathrm{EN}_3(t)^{\vartheta_2}\right)^{1/\vartheta_2}$$

where $\sum_1^3 \kappa_i = 1$ and "*parameter $\vartheta_2 < 1$ determines the elasticity of substitution between different energy sources, and $\kappa_i$ measures the relative energy efficiency of the different energy sources*" (Golosov *et al.*, 2014). Technical changes are endogenous in this model.

cost for the non-energy sectors[94]. In terms of results, this model suggested a higher tax than the seminal DICE. Indeed, Nordhaus SCC in 2015 is approximatively 30\$/tonne of coal compared to 57\$/tone for Golosov *et al.* (2014) DSGE.

To focus on the financial dimension, Benedetti *et al.* (2019) introduced a way to channel the potential impacts of carbon pricing on equity prices in order to propose an optimal portfolio construction under transition risk. They introduced the carbon tax, $\tau_c$, implemented at time $T$ with a probability $\pi$ and assessed its effect on price equilibrium. At each date, a switch can occur, between pre-tax equilibrium $(p^*, q^*)$ and post-tax equilibrium $(\bar{p}^*, \bar{q}^*)$ affecting supply and demand for the resource[95]. The revenue generated by the reserve after carbon pricing is null, if the exploitation costs (extraction, transportation, storage, etc.) exceed the new mark price and decrease otherwise (Bennedetti *et al.*, 2019)[96].

Policy makers, financial practitioners and the IPCC used a range of advanced models to develop scenarios[97]. For instance, the MIT-ISGM is a model resulting from the association of the MIT Economic Projection and Policy Analysis (EPPA)[98] and MIT Earth System Model (MESM). This model therefore couples complex nesting structure with preferences for final consumption and productions. It is a computable general equilibrium model that allows non-homothetic preferences[99], with a quite complex atmosphere-ocean global circulation model. Similarly to AIM/Dynamic-Global (Masui *et al.*, 2006) it introduces Armington assumption extending the standard modeling to macroeconomic problems, and inter-regional trade-flows with preferences toward domestic vs. foreign productions varying according to the sector and region. However, this model is an evaluation model, answering so-called '*what if*' questions. It does not allow us to determine the "*optimal policy or to endogenously simulate other behavior of the political actors in the face of economic and environmental change*" (Chen *et al.*, 2015). This type of model can be used in asset pricing

[94]To give an overview of the general equilibrium problem posed by Golosov *et al.* (2014) we have the profits defined for the two types of firm posed as follows:

$$\begin{aligned}
\max_{\mathrm{EN},K,L} \Pi_{\mathrm{prod}}(t) &= \mathbb{E}_0\left[\sum_{t=0}^{\infty} q(t)\left((1-\tau_i(t))\mathrm{EN}_i(t) - rK_i(t) - w_t L_i(t) - \sum_{j=0}^{n_{\mathrm{EN}}} p_j(t)\,\mathrm{EN}_{i,j}(t)\right)\right] \\
\max_{Y(t)} \Pi_{\mathrm{cons}}(t) &= \mathbb{E}_0\left[\sum_{t=0}^{\infty} Y(t) - rK_i(t) - w(t)\,L_i(t) - \sum_{i=0}^{n_{\mathrm{EN}}} p_i(t)\,\mathrm{EN}_{0,i}(t)\right]
\end{aligned}$$

where energy producers maximizes their sales of energy $i$, $q(t)$ is the quantity sold, $(1-\tau_i)$ is the remaining fraction after tax, minus its general expenditures and the cost of energy $j$ consumed for the production of $i$. Thus, the model accounts for intermediary consumption. On the other hand, the firm consuming simply maximizes its output given its costs, including the energy supply. This type of modeling, coupled with a nested structure, where the elasticities would be frequently updated, can be used to identify transition pathways within the energy sector. Household utility and market clearing must be added to this relationship, as mentioned Section 2.1.3 on page 16.

[95]Then, they used '*company-specific forward production curves*' (from Carbon Tracker) to determine the impact of the new equilibrium on companies revenues.

[96]While Golosov *et al.* (2014) targeted the optimal tax in a general equilibrium, this study approaches the exposure at risk of stranding the reserves RSV $(t)$ extraction process from a portfolio construction standpoint (see Figure 10 on page 47).

[97]Some of the most famous scenarios are given in Appendix A.2 on page 68.

[98]EPPA was developed by Johan Reilly (MIT) and was used in China and Mexico to compute their national determined contribution (NDC).

[99]Heterogeneous non-monotonic utility functions. It can explain different pattern of consumption in different countries.

to assist managers in both short- and medium-term decisions[100] if the behavior of agents is representative enough in the situation of interest. The functioning of economic equilibrium of this model is developed in Paltsev *et al.* (2005).

The IMACLIM-R model developed by the CIRED is neither a macro nor a CGE strictly speaking. This model is a recursive general equilibrium that combines an annual static macroeconomic equilibrium specification[101], and a moving envelope for technical possibilities. This way, this advanced model embeds short- medium- and long-term constraints:

> "*while the static equilibrium ensures economic consistency between all flows and relative prices under short-run constraints at each point of time, the dynamic components represent the shift across time of these technology, equipment stock and endowment constraints.*" (Hourcade *et al.* 2015, page 18).

The growth drivers for the economy are the traditional ones (demographic, capital, factor of productivity, and saving rates) but the model embeds specific definitions for a fossil fuel depletion, electricity generation, residential energy end-uses, transportation and agriculture industry and services modules. The interaction between specific modules and the static economic equilibrium allows us to introduce plausible scenarios. The uncertainty of the output raises with the complexity of the model for numerous identified reasons, therefore they identified seven key parameters that are oil and gas markets, Middle East strategy, coal markets, alternative liquid fuels supply, carbon-free option, energy end-uses technologies and development patterns.

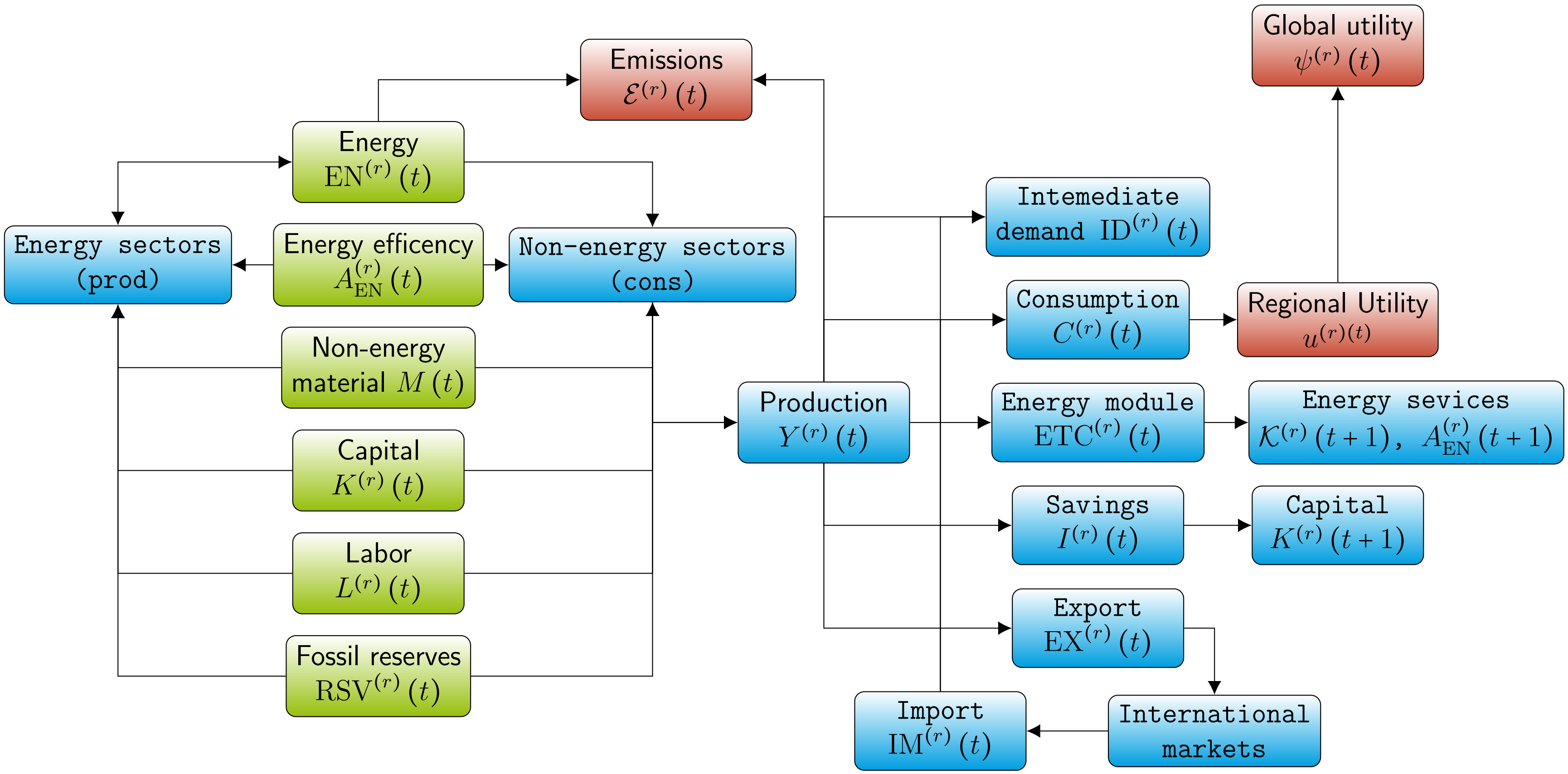
These models give complex representation of the world and can be used either to project coherent and plausible scenarios or to perform academic overarching critical thinking challenges. IMACLIM-R model can be associated to others, to encompass a better description of, for example, the energy module and life-cycle effects. It was indeed associated to the TIAM-FR[102] model in a soft-linking experiment between top-down and bottom-up models (Assoumou *et al.*, 2017). Financing sustainable scenarios and building smart aligned portfolios could require this type of association, thus introducing a financial module about which most models are silent so far.

[100] The notions of short, medium and long can appear rather vague. The idea is that allowing more financial variables, such as prices, to enter the model allows us to anticipate more plausible outcomes. Therefore, this type of modeling allows us, while tracking a long-term objective, to make the optimal decision with respect to the market short- and medium-term constraints.

[101] Based on 34 equations in the specification given by Cassen *et al.* (2010).

[102] The time integrated assessment model (TIAM) describes the global energy system with a high level of disaggregation on energy sources, technologies, and end uses (Boubault *et al.*, 2018).

Figure 10: Example of a modular structure of a global dynamic model

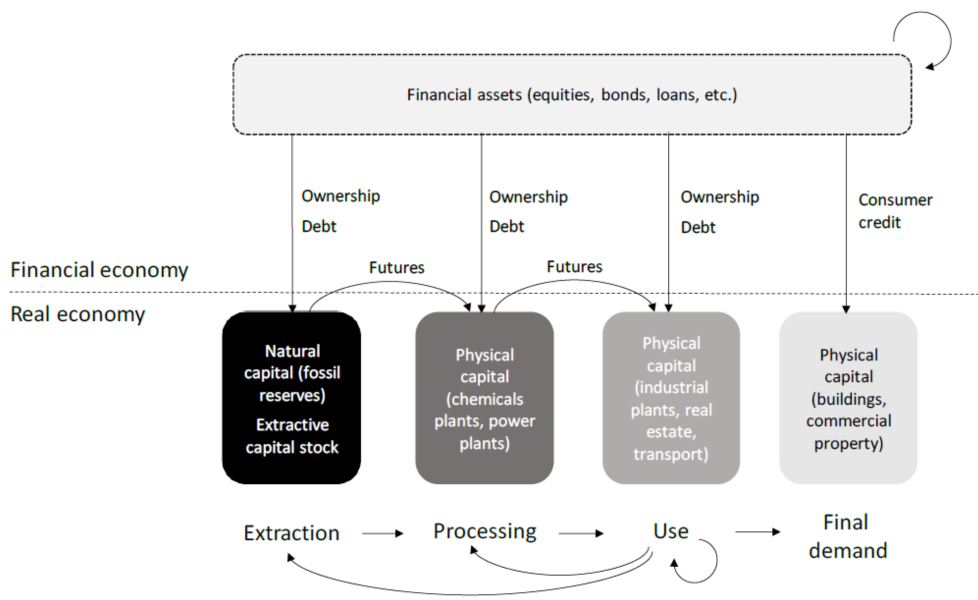



- The green modules are examples of economic inputs (non-exhaustive). The uncertainty relative to these modules comes from measuring standards and assumptions.
- The blue modules represent processes. They depends on model uncertainty (See Section 3.4 on page 50).
- The red boxes represent the control variables. These variable are uncertain because hardly measurable. Indeed, utility can be a rather qualitative concept and payer contributions to the overall emission are not properly measured yet.

### 3.3.2 Financial stress-test and network effects

From a financial perspective, risk assessment models are either statistical measures such as $VAR$[103] when historical data are available, or so-called stress-test models. The latter are more suitable for measuring climate risks. They provide instant information evaluating the impact of a pre-defined stress on the modeled system. In addition, they are particularly useful for assessing the undervaluation of these risks due to the omissions of interdependencies. Campiglio *et al.* (2017) and Cahen-Fourot (2019) have made an important contribution to the literature on this subject. The two papers focus on the cascading effects of the risk of the physical capital stranding[104]. The initial hypothesis is that some resources will become stranded and that the losses will be endogenously diffused between non-financial actors as shown by the diagram in Figure 11. This procedure allows us to assess the capital stock at risk of stranding implied by decarbonization and its implication on the entire network.

Figure 11: Natural, physical and financial assets at risk of stranding




Source: Campiglio *et al.* (2017).

In these two papers, the sector classification used is the NACE[105]. The cascading effects of the physical risk of stranding were shown to be important and sector-specific:

> "*We show how, in a sample of ten European countries, mining is among the sectors with the highest external asset stranding multipliers. The sectors most affected by capital*

[103]See vector autoregression in the glossary.

[104] Models assessing this effect are however not integrated assessment models but stress-testing frameworks which are more commonly used, in the field of finance.

[105]The term NACE is derived from the French ***N**omenclature statistique des **A**ctivités économiques dans la **C**ommunauté **E**uropéenne*. Various versions have been developed since 1970. Table 6 on page 78 gives the first sectoral level of the NACE nomenclature. For instance, sector A is agriculture, sector B is mining and quarrying and so on until U. However, their studies consider the NACE sectors as far as S.

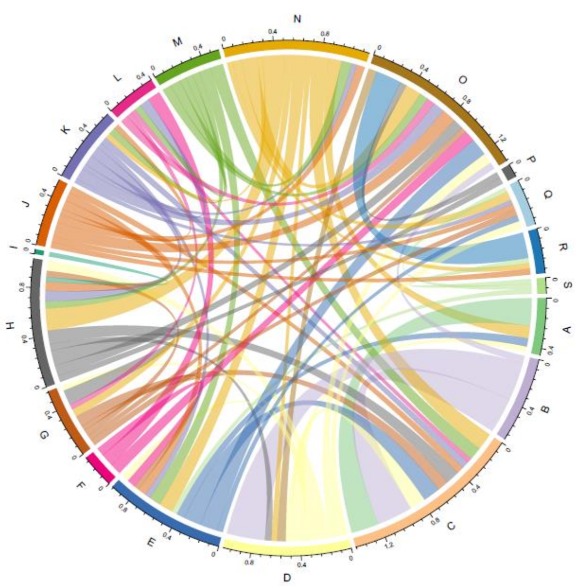
*stranding triggered by decarbonization include electricity and gas; coke and refined petroleum products; basic metals; and transportation. From these sectors, stranding would frequently cascade down to chemicals; metal products; motor vehicles; water and waste services; wholesale and retail trade; and public administration*" (Cahen-Fourot *et al.*, 2019, page 2).

Formally, the authors introduced $S$, the matrix of asset stranding multipliers:

$$S = \hat{\kappa} G^{\intercal}$$

where $G$ is the Ghosh[106] matrix and $\hat{\kappa}$ is the sectoral capital intensity vector, the model is able to deduce the projection on each sector of the stranding consequences of the decarbonization objectives. Graphically, Cahen-Fourot *et al.* (2019) gave the chord diagram presented in Figure 12 where we can observe the respective projections of the interdependencies between sectors. For instance, we can read that the public and administrative sector O does not affect any other sector, but is however affected by many of them. On the other hand, the mining sector B, is directly affected by the stranding risks and reports its risks on the manufacturing sector C, and the electricity sector D.

Figure 12: Chord diagram of S minimal fully-connected network for Germany (2010)



The NACE nomenclature to read this chart is given in Table 6 on page 78.
Source: Cahen-Fourot *et al.* (2019).

These recent works revealed the interconnectedness of the real economy businesses and their dependency. A similar study, this time focusing on the financial assets[107] at risks of stranding,

[106] Extending the concept of Leontief Input-output matrix. See NAMEA in the glossary.
[107] The first-round exposures of the financial actor $i$ toward equity, bonds and loans were computed using:

$$O_i = \sum_{s \subset \mathfrak{S}_{\mathrm{CR}}} \sum_{j \in s} \left( \alpha_{i,j}^{\mathrm{Equity}} + \alpha_{i,j}^{\mathrm{Bond}} + \alpha_{i,j}^{\mathrm{Loan}} \right) + R_i$$

and therefore subject to default has been published by Battiston *et al.*, (2017). In particular, this paper introduces an indirect effect due to the inter-dependency caused by interbank connections:

> "*Using empirical data of the Euro Area, we show that while direct exposures to the fossil fuel sector are small (3-12%), the combined exposures to climate-policy relevant sectors are large (40-54%), heterogeneous, and possibly amplified by indirect exposures via financial counterparties (30-40%)*" (Battiston *et al.*, 2017, page 1)[108].

The interconnectedness of the global system is obvious and clearly exposed by these studies. Climate related risks are therefore '*everywhere*'. In order to be able to assess financial actors and portfolios exposures, it is necessary to introduce new climate accounting methodologies to track *emissions induced*[109] by financial activities. The main purpose is to generate climate indicators that are now drowned in the information flow leading to the construction of ESG scores. For instance, Rose (2014) has proposed an *account-by-issue*[110] accounting method to better identify impacting actors and levers of action to influence the climate. This form of accounting has also been developed in order to be simply applicable to emission induced by financial allocations, the resulting method is called P9XCA[111]. Other metrics can be considered[112] from the combination of different elements introduced in this review (targets obtained using optimization models, minimization of systematic or network risks, statistical trends and vector autoregressions, etc.). All in all, we can observe that these models and stress-testing frameworks all provide information on the possible evolution of macroeconomic and climate variables that are conditioned by the varying modeling standards. Before using the output of these models as an input in investment processes, it is important to assess the uncertainty related to their structures.

## 3.4 Model uncertainty

The model uncertainty refers to the uncertainty of the results due to the multiplicity of modeling choices. Indeed, the previous part demonstrated the existence of many ways to embed endogenous or induced technical change in production functions or to model and project to potential damage on the economy, sectors or assets. In a forward-looking environment, therefore, the legitimacy of the models is based on the uncertainty relative to their functional structure and on the relevance of the calculus performed and approximation made.

---

where $O_i$ is the total outstanding amount of assets of the financial actor $i$, $\alpha_{i,j}$'s denote monetary values of the exposure of $i$ to the securities associated with economic actor $j$, $R_i$ is the residual accounting for other instruments and $\mathfrak{S}_{\mathrm{CR}}$ the set of climate-relevant sectors according to Battinston *et al.* (2017). The second-round exposure of the same financial actor $i$ also considers its exposure toward other financial actors $i$ that are also exposed. Is defined as follows:

$$O_i = \left( \sum_{j \in \mathfrak{F}} \alpha_{i,j}^{\text{Equity}}\left(O_j\right) + \alpha_{i,j}^{\text{Bond}}\left(O_j\right) + \alpha_{i,j}^{\text{Loan}}\left(O_j\right) \right) + \left( \sum_{k \in \mathfrak{S}/\mathfrak{F}} \alpha_{i,j}^{\text{Equity}} + \alpha_{i,j}^{\text{Bond}} + \alpha_{i,j}^{\text{Loan}} \right) + R_i$$

where $\mathfrak{F}$ denotes the set of financial institutional sectors.

[108] Quoted from the online version, the abstract in nature page 283 has been changed.

[109] See induced emissions in the glossary.

[110] See account-by-issue in the glossary.

[111] See Appendix A.4 on page 72.

[112] For instance the NEC, for net environmental contribution, that is an open source initiative to track environmental impact of actors: https://quantis-intl.com/net-environmental-contribution/.

Table 4: Classification of Integrated Models

| | **Technological detail** | |
|---|---|---|
| Calculus | *Top-down* | *Bottom-up* |
| **Welfare maximization** | Optimal Growth models<br>DICE<br>MERGE<br>FEEM-RICE<br>ENTICE-BR<br>DEMETER<br>AIM/Dynamic-Global | |
| **Cost minimizing** | | Energy System Models<br>MESSAGE<br>GET-LFL<br>DNE21+ |
| **Initial value problems** | Simulation models<br>E3MG | |
| **Static Equilibrium + Recursive dynamics** | Computational General Equilibrium<br>IMACLIM-R | |

Source: Edenhofer *et al.* (2006).

Edenhofer *et al.* (2006) compared the outputs of each model categorized in Table 4 with their provided parametrization. These models are all deterministic. One could add DSGE models since Golosov *et al.* (2014) conducted a study on the optimal price of carbon in a decentralized general equilibrium with stochastic variables, which had little influence in this model[113]: "*specifically, the stochastic values of future output, consumption, and the stock of $CO_2$ in the atmosphere all disappear from the formula" (Golosov et al., 2014, page 41-42).* To assess the model uncertainty of the most recent IAMs, Gillingham *et al.* (2015) used a Monte-Carlo approach on the input probability density function and with several model functionals[114]:

$$Y^m = H^m(z, \alpha, u)$$

This study provides for a range of uncertain inputs, entered into several assessment models to produce the distribution of the outputs. In particular, they picked three uncertain parameters of interest namely (i) the *rates of growth population*, (ii) the *rate of growth of productivity* and (iii) the *climate sensitivity*[115]. If we rely on the Nordhaus framework, the two first variables correspond to the socioeconomic module and the third represents the uncertainty within the climate module. This study therefore explored to some extent both structural and parameter uncertainty.

We note that temperature distributions are similar in every model. They all spread between 1.75°C and 7.33°C and are centered around 3.24°C for the baseline scenario. Moreover, for 95% of

[113]Note that the targeted value was the optimal tax, the influence of stochastic variable can be more significant when computing optimal allocation strategies or to assess assets specific climate risks.

[114]The functional of the following models: DICE v2014; FUND v2014; GCAM, 2011; MERGE v2014; MIT IGSM v2015 and WITCH 2014.

[115]See equilibrium climate sensitivity in the glossary.

Table 5: Distribution of temperature change in the Baseline case 2100 (°C)

| Temperature | 0.1% | 5% | 25% | 50% | 75% | 95% | 99% | 99.9% |
|---|---|---|---|---|---|---|---|---|
| DICE | 1.60 | 2.38 | 3.12 | 3.76 | 4.51 | 5.80 | 6.88 | 8.28 |
| FUND | 1.96 | 2.63 | 3.19 | 3.19 | 4.17 | 5.12 | 5.92 | 6.96 |
| CGAM | 1.59 | 2.46 | 3.23 | 3.23 | 4.56 | 5.73 | 6.64 | 7.79 |
| MIT-IGSM | 1.30 | 2.31 | 3.05 | 3.05 | 4.13 | 4.97 | 5.58 | 6.29 |
| MERGE | 2.20 | 2.93 | 3.61 | 3.61 | 4.90 | 6.12 | 7.13 | 8.46 |
| WITCH | 1.83 | 2.60 | 2.82 | 3.22 | 4.23 | 5.01 | 5.58 | 6.22 |
| Average | 1.75 | 2.55 | 2.79 | 3.24 | 4.42 | 5.46 | 6.29 | 7.33 |

Source: Gillingham *et al.* (2015).

the runs, temperatures go beyond 2.55°C if nothing *more*[116] is done. The constrained optimization of the utility post-2016 does not allow us to align on a 2°C trajectory which is in line with these results. The common approach to reducing modeling uncertainty is to eliminate possible structures when they do not match empirical observations. However, this mismatch can be due to parameter uncertainty and modelers are more likely to introduce update parameters than change the entire structure of their model. If we simply wish to observe temperature distributions of the business-as-usual case (Table 5) and deduce the value-at-climate-risk of financial positions, the '*Ockham razor*' principle would advocate for a '*DICE-like*' modeling standard. The definition of the transmission channels between expected change in average temperatures, business cycles and asset prices would require further assessment.

To focus on the carbon pricing dimension, results are in fact to be interpreted differently according to the type of modeling. Optimal growth models give the optimal price where the social cost of carbon $\mathrm{SCC}\,(\mathrm{t})$ is the price maximizing the welfare over time. The shadow price is explicitly the optimal price of carbon given by the model (as it was for DICE). In optimal growth models, we can distinguish two sub-categories. The first-best models of the economy give a Pareto efficient solution, with no market imperfection and, thus, the shadow prices of carbon will be the social price. The second-best[117] optimums do not generally consider a carbon tax strictly speaking. They simulate the market with imperfections and distortions. The optimal price of carbon is the price that ensures stabilization with minimum welfare losses. The shadow price obtained by energy system models is computed by optimization of the energy sector only. The calculus of this price omits feedback effects between macroeconomics and the energy sector. In some other models, there is no optimization of the welfare[118] and the optimal tax represents only the required one to achieve stabilization regardless of its effects on consumption (IMACLIM-R). The introduction of induced technological change approximatively halves the optimal taxes for the models concerned but has less effect on shadow prices for the others. The model comparison by Edenhofer *et al.* (2006) highlighted that results were largely determined by: baseline effects,

[116]In most contemporary models the baseline has become an extension of the currently implemented regulations and policies. It is the projected evolution of the current climate action trends.

[117]The second-best model refers to a situation where at least one of the optimality conditions cannot be solved.

[118]This choice can be made when modelers '*refuse*' to use highly hypothetic damage functions (see Section 3.1 on page 34). Consequently, they cannot model the trade-off to balance future losses, that cannot be quantified. These models generally focus on the adaptation pathways required to maintain temperatures bellow 2 or 1.5°C.

first-best or second-best assumptions, model structure, long term investment decision backstop and end-of-the-pipe technologies. The notion of carbon price is consequently a notion to put into perspective with respect to the model used to determine the optimal path.

## 3.5 Shared socioeconomic pathways

The varying modeling standards and the introduction of endogenous technical change (ETC) for cleaner energy does not bring any enlightenment on how to concretely invest and take action to reduce climate risks. The models to assess from forward-looking standpoint the evolution of productive economic systems have been developed but the concrete means to either reduce negative climate impacts or adapt to face the consequences of human activity are to explore. The set of plausible scenarios have been defined in the literature and the metrics to assess the exposure with respect to these scenarios partly approached[119].

The models are built to project the trends of variables based on economic and political assumptions. It is likely that since the first trajectories were published, the following were, intentionally or not, calibrated on the same pathways. The question becomes, what would be the answer if we slightly change the formulation of the problem or if the political environment was to change? This question has been approached quite pragmatically by practitioners. Conscious of the impossibility of constructing a model that projects or predicts the future, some choose to discuss plausible scenarios based on socioeconomic and policy assumptions. The answers of the system under these assumptions are based on observed behavioral responses given by society. The resulting pathways are called the '*shared socioeconomic pathways*' (SSP). These scenarios are simplified causal responses of the socioeconomic system to the variation of two main dimensions: adaptation vs. mitigation. Mitigation refers to the capacity to shift quickly toward a greener economy and consumption while adaptation corresponds to the society's capacity to adapt to, or at least deal with, climate change. The latter is the dimension that is most subject to uncertainty and therefore either ignored or under-evaluated by many assessment models[120].

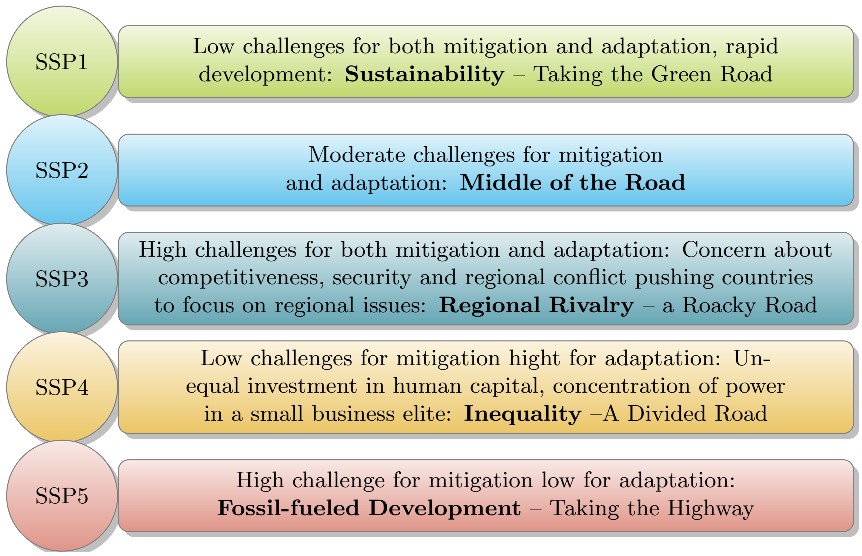
SSP1 an SSP2 are the scenarios that describe a relatively easy transition toward a sustainable economy while SSP3 represents a closing economy where internal issues force the rise of extremes and nationalism. The SSP4 is a representation of a multi-level society. We see here that human capital[121] is introduced and plays an essential role in these scenarios. The Taking the Highway SSP5 translates a high resiliency of the socioeconomic environment toward climate change and, consequently, nothing is done to reduce negatives as the impacts are limited by the system's adaptability. SSP3 with the rise of worldwide nationalism is the second worst in terms of temperatures. It also goes with a more rural and growing population as demonstrated Figure 14, and the worst economic scenario in terms of GDP because of the closing of trade roads and high protectionist taxes. SSP5 keeps a constant economic growth rate despite a reduction of population which might also be due to high urbanization implying, for instance, a higher *cost of raising a child* in cities. Defining the investment process as a representative game theory system, where agents choose

[119] See Appendix A.2 on page 68 to see the varying scenario proposed as input for financial actors.

[120] In the last specification for the WITCH model the damage function embedded an adaptation factor limiting the damage. This factor was defined as a function of R&D spending (stock of knowledge) and human capital (or education) (Emmerling *et al.*, 2016).

[121] See Section 2.1.1 on page 12.

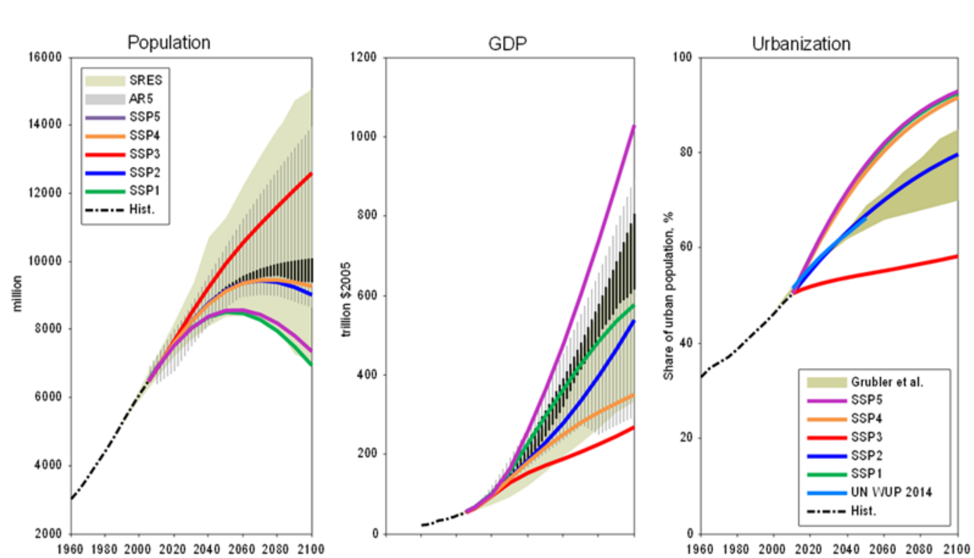
Figure 13: The shared socioeconomic pathways




Source: O'Neill *et al.* (2017).

between, in this case, five possible future states, investing under climate constraints becomes a matter of choosing the most likely or preferable future. Acquiring a two-dimensional view of each issuer's behavior and placing investors' portfolios could help to both reveal the current picture of wide range of beliefs and to advocate for changes in portfolio positions when possible.

Figure 14: Projections of population, economic growth and urbanization across SSP




Source: Riahi *et al.* (2017).

These scenarios are built from the IAMs that were considered by the *intergovernmental panel on climate change*[122] (IPCC) as the most scientifically legitimate and relevant. However, they remain only representative path-dependent plausible outcomes; the subject has demonstrated in this review that there is uncertainty at every level. The results also suggest focusing on the transition pathways, that allow investors, politicians or non-governmental organizations to act in favor or against one of these scenarios. Therefore, developing SSP-based solutions would be an interesting to reduce the dimensionality of the problem.

# 4 Conclusion

Even the most complex integrated assessment models provide highly schematic patterns and one can question the representativeness of the projected scenarios. They are indeed often criticized for bringing illusory scientific legitimacy to projections based on numerous assumptions (Pindyck, 2017). Modelers themselves usually present these models and results as highly hypothetic. These prior uncertainties in the socioeconomic scenarios, that are usually used as the input to generate climate indicators by financial actors, convolute the assessment of financial risks implied by climate change. From a strictly financial standpoint, determining for instance issuers' risk of stranding is a non-trivial task either and the current stress-testing frameworks rely on input-output models with substitution scheme that are still unclear.

Even if they are not designed to provide a robust estimation of the future, IAMs present a rather representative idea of the efforts that are required to keep the temperatures below a certain level. Despite these results advocating mostly for immediate reduction of greenhouse gas[123] emissions, policy makers are having difficulties effectively implementing carbon pricing systems. Indeed, the non-uniform implementation of carbon taxes[124] will either reduce local corporate competitiveness or representative households' purchasing power. This in turn will affect affecting political stability and, therefore, potentially GDP as demonstrated by recent events in France. Moreover, the relatively short presidential terms and the increasing randomness of electoral processes have highlighted the lack of viability of global agreements in the long run as the political environment is subject to a change of leadership, among other potential political and economic disturbances. Therefore, computing the optimal global carbon price seems to have little impact as both consumption and GHG emissions are following the '*tragedy of the commons*' systemic archetype with no commonly accepted and long-lasting authority able to force the current generation to fix what will become the inherited sentence of the next. On the other hand, central banks are becoming aware of the no-way-out situation implied by this systemic archetype, and declared themselves ready to take action to preserve the environment. This would certainly support the likely introduction of carbon emissions regulations. If they do, their statutes might allow them to implement effective measures as they are not directly subject to elections[125].

[122] See IPCC in the glossary.

[123] With some more than others (Stern, 2007). We reiterate that the main influential parameters according which the optimal carbon price vary are the discounting factor, population, endogenous technical changes and the expected damage elasticity.

[124] Respectively the implementation of an emission trading system (ETS) faces the same constraints: they both require a certain level of homogeneity in beliefs and in willingness to pay.

[125] Their power is limited in the case of a more nationalist pathway such as the environment described in the

Despite these numerous pitfalls, portfolio decarbonization methodologies[126], that are based on the scenarios proposed by optimization models, are a consistent first step to reduce transition risks (or carbon risks) exposure. They also participate in increasing the demand for greener assets thus acting on issuers' cost of capital. This dual objective of responsible investment can only be achieved with a certain level of global commitment from investors. The change in trend appears to be in favor of responsible investors, as demonstrated by recent papers exhibiting an increasing interest in responsible investing. Moreover, the integration of extra-financial criteria in investment strategies appears to be increasingly rewarded as well. The impact of ESG on supply and demand has already being channeled to the equity market and directly impacted share prices (Bennani *et al.* 2018).

ESG scores are aggregated metrics that are not representative of climate risks and climate relevant indicators must be developed to properly apprehend climate risks from a financial standpoint. More generally, the introduction of allocation processes based on climate and social risks is now explicitly required. Indeed, it was noted by Diaz-Rainey *et al.* (2017) that elite financial journals have largely ignored climate change and both optimizers and pricing tools are not adapted to track climate variables. In practice, the ability to implement the solutions suggested by climate models is so far limited to best-in-class strategies based on perfectible data sources[127]. Current climate metrics are generally obtained by computing the standard deviation of the issuer reduction target with respect to the snapshot of required reduction published by the most recent IEA report. These best-in-class strategies, currently applied in ESG scoring system, are most likely the best concrete and relevant tool we have so far to implement responsible strategies, however a continuous time integration of both targets and effective reduction would allow more flexible processes. In addition, if the best way to finance transition is likely to be through the bond market, for example, investing in green bonds[128] or infra funds and making issuing agents able to cover their respective abatement costs[129], then the optimal portfolio allocation strategy on a diversified universe has to be defined. This could lead to the level of global engagement required, despite heterogeneity of beliefs, to align temperature on an optimal and realistic path.

Now that the semantic and mathematical concepts have been clarified, the challenge is to develop an interactive financial modeling framework that provides answers to practical questions. The assessment of financial risks can be based on the combination of the varying structures presented in this paper. The main challenge is to construct a normative framework to compare investors portfolios with respect to climate relevant criteria. The common practice is to construct financial factors allowing to channel the extra-financial information in the pricing system. The

SSP3.

[126]The construction of low-carbon portfolios is already proposed on the basis of declarative information or metrics issued by external providers. Moreover, it has been shown that passive long-term strategy could be decarbonized without sacrificing financial returns (Anderson *et al.*, 2016).

[127]See portfolio alignment methodologies in Appendix A.3 on page 70.

[128]Which is still a very limited option considering the extremely narrow market share they represent. See green bonds in the glossary.

[129]We reiterate that models track the optimal trade-off between abatement and future damage. If damage can not be precisely assessed, abatement cost could quantitatively enter as an additional information in an optimization framework. However, asset managers and thus investors have so far no direct access to detailed capital expenditures related to reducing negative environmental impacts. Therefore, both the required cost and effective abatement are ignored by the finance community.

construction of risk portfolios (Fama-French, 1992) allow us to rank securities according to one property of interest and to identify the characteristics priced by the market. The construction of smart carbon or transition portfolios become therefore an operational issue once we have confidence in the data used as input. Some more advanced factors could be developed to embed the social dimension that is indissociable from environmental constraints.

The integration of these multifaceted climate models go further than simply labeling investors portfolios and monitoring plausible pathways implied by the activities they finance. Indeed, it has been shown that statistical and quantitative financial models are used to project short-term trends. International general equilibrium models can provide short- medium- and long-term responses to stimuli of interest. Welfare optimization models enable long-term objectives to be tracked. One could also embed machine learning modules to constantly reassess the scale of the responses of the dynamic equilibrium models. The proper combination of the models and metrics in the field of asset management could introduce complementary operators providing additional insights in the decision-making process and generating out-performance during difficult times.

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# A Appendix

## A.1 Notations

| | Indices | |
|---|---|---|
| $\mathfrak{E}$ | Representative economy | 17 |
| $\mathfrak{S}$ | Firm (or representative sector) | 16 |
| $\mathfrak{S}_{\mathrm{CR}}$ | Climate relevant sector | 49 |
| $\mathfrak{F}$ | Financial sector | 49 |
| $\mathfrak{H}$ | Household (active economical unit) | 17 |
| $m$ | Possible modeling specification | 51 |
| $t$ | Time variable | 9 |
| $r \in \mathfrak{R}$ | Region | 32 |
| $\{\mathrm{AT}, \mathrm{UP}, \mathrm{LO}\} \in \mathfrak{L}_{\mathcal{C}}$ | Set of geophysical layers for concentrations (atmospheric, upper ocean and biosphere, deep ocean) | 22 |
| $\{\mathrm{AT}, \mathrm{LO}\} \in \mathfrak{L}_{\mathcal{T}}$ | Set of geophysical layers for temperature (atmospheric, upper ocean and biosphere) | 22 |
| | **Parameters** | |
| $A_0$ | Initial knowledge factor | 11 |
| $a_1$ | Damage function scale parameter (DICE) | 21 |
| $a_2$ | Damage function scale parameter quadratic (DICE) | 21 |
| $\alpha_{\mathrm{EN}}$ | ENTICE substitution scale parameter | 40 |
| $\alpha_{\mathrm{FOS}}$ | ENTICE scaling factor | 40 |
| $\alpha_{\mathrm{ES}}$ | CES coefficient energy services (WITCH) | 42 |
| $\alpha$ | Elasticity capital–effective labor | 11 |
| $b_1$ | Abatement cost scale parameter | 22 |
| $b_2$ | Abatement cost exponent parameter | 22 |
| $\beta$ | Elasticity human capital–effective labor | 12 |
| $C_{\mathrm{AT}}$ | Atmospheric thermal capacity | 25 |
| $C_{\mathrm{LO}}$ | Near surface thermal capacity | 25 |
| $\Delta$ | Time step (for discrete models) | 23 |
| $\delta_{N_c}$ | Natural capital depreciation rate | 13 |
| $\delta_K$ | Depreciation for capital | 11 |
| $\delta_{g_A}$ | Decline of rate of knowledge growth (DICE) | 20 |
| $\delta_{g_L}$ | Decline of labor force growth (DICE) | 20 |
| $g_A$ | Technical growth rate | 11 |
| $g_K$ | Capital growth rate | 13 |
| $g_L$ | Population growth rate | 11 |
| $g_Y$ | Economic growth rate | 13 |
| $L_0$ | Initial labor input | 11 |
| $\vartheta$ | Elasticity natural capital–effective labor | 13 |
| $\theta$ | Risk aversion (CRRA) | 14 |
| $\rho$ | Inter-temporal substitution parameter | 14 |
| $\phi_{i,j}$ | Temperature diffusion matrix $i, j \in \mathfrak{L}_{\mathcal{T}}$ over period $\Delta$ | |
| $\xi_{i,j}$ | Concentration diffusion matrix$i, j \in \mathfrak{L}_{\mathcal{C}}$ over period $\Delta$ | |

| Variables | | |
|---|---|---|
| $A(t)$ | Knowledge or technical progress Harrod-neutral factor | 9 |
| $A_{\mathrm{TFP}}(t)$ | Total factor of productivity Hicks-neutral | 11 |
| $A_{\mathrm{EN}}(t)$ | Energy technological progress or energy efficiency | 38 |
| $\mathrm{B}(t)$ | Backstop technology | 39 |
| $C(t)$ | Aggregated consumption | 14 |
| $c(t)$ | Consumption per capita | 14 |
| $\mathcal{C}(t)$ | Concentration vector | 22 |
| $\mathcal{C}_{\mathrm{AT}}(t)$ | Concentration of GHG in the atmosphere | 22 |
| $\mathcal{C}_{\mathrm{UP}}(t)$ | Concentration of GHG in the upper ocean | 22 |
| $\mathcal{C}_{\mathrm{LO}}(t)$ | Concentration of GHG in the deep ocean | 22 |
| $D(t)$ | Damage functions | 21 |
| $e(t)$ | Household endowment | 17 |
| $\mathcal{E}(t)$ | Total emissions | 22 |
| $\mathcal{E}_{\mathrm{Land}}(t)$ | Other emissions | 22 |
| $\mathrm{EL}(t)$ | Electric energy input | 40 |
| $\mathrm{EN}_{\mathrm{FOS}}(t)$ | Fossil energy | 39 |
| $\mathrm{EN}(t)$ | Energy composite | 39 |
| $\mathrm{EN}_{\mathrm{RNW}}(t)$ | Renewable energy | 39 |
| $\mathrm{EN}_{\mathrm{FOS}}(t)$ | Fossil energy | 39 |
| $\mathrm{EX}(t)$ | Export | 47 |
| $F(t)$ | Production function | 9 |
| $f(t)$ | Reduced form production function | 10 |
| $\mathcal{F}_{\mathrm{EX}}(t)$ | Exogenous Forcing | 23 |
| $\mathcal{F}_{\mathrm{RAD}}(t)$ | Radiative Forcing | 23 |
| $G$ | Ghosh matrix | 49 |
| $H(t)$ | Human capital | 12 |
| $I(t)$ | Investment | 20 |
| $\mathrm{IM}(t)$ | Imports | 47 |
| $\mathrm{ID}(t)$ | Intermediate demand | 47 |
| $K(t)$ | Capital stock | 9 |
| $\mathcal{K}(t)$ | Knowledge stock | 41 |
| $L(t)$ | Aggregated labor input | 9 |
| $\Lambda(t)$ | Abatement cost function | 21 |
| $\mu(t)$ | Mitigation ration | 22 |
| $M(t)$ | Non-energy materials | 47 |
| $N_c(t)$ | Natural capital | 13 |
| $\mathrm{NE}(t)$ | Non-electric energy input | 40 |
| $O$ | Total outstanding exposure of financial actor | 50 |
| $\Omega_{\mathrm{climate}}(t)$ | Climate related loss coefficient | 13 |
| $\Omega_D(t)$ | Climate related loss coefficient (damages only) | 21 |
| $p(t)$ | Prices vector | 17 |
| $\Psi_{i,j}(t)$ | Matrix of positions household $j$ portfolio in Firm $i$ | 17 |
| $Q(t)$ | Aggregated production: net output | 19 |
| $r(t)$ | Interest rates | 16 |
| $\mathrm{RSV}(t)$ | Fuel Reserves | 47 |

## A.2 Plausible scenarios

### A.2.1 Energy mix scenarios

As energy supply is the most carbon intensive sector, and more generally as energy represents the largest portion of GHG emissions within each sector, the most common way to reduce global emissions is to impose a global shift in the energy supply side. To encourage this shifting the international energy agency (IEA), and others, have developed optimal energy mixed pathways with respect to a range of assumptions.

**IEA scenarios** They are the input for most of the currently commercialized solutions to build 2°C portfolio methodologies based on the energy mix transition. These scenarios are not a forecast of the future but explore the different possibilities across the energy system following different pathways which are highly sensitive to government decisions.

- Current policies scenario is the IEA reference or baseline scenario.
- The new policies scenario (NPS) corresponds to an increase in energy demand, urbanization in developing countries, in a context where today's ambitious policies are implemented on the energy sector.
- The sustainable development scenario (SDS) is similar to the implications of the SSP1 and represents an integrated approach to achieve international objectives on climate change.
- 2°C scenario (2DS) and Beyond 2°C scenario (B2DS) are optimistic visions to achieve the 2°C scenario.
- Energy technology perspectives (ETP) 2 degrees scenario
  ETP 2°C scenario (ETP-2DS) and beyond 2°C scenario (ETP-B2DS) are similar to the previous ones in terms of targets but does not depend on the appearance of unknown backstop technologies. All technology options are already available or at a stage of development that makes commercial-scale deployment possible within the scenario period.

The IEA disclosed a wide range of scenarios[130] developed with the world energy model (WEM). The examples given are the *future is electric* scenario, the *faster transition* scenario, the *low oil price* case, the *energy for all* case, the *450 scenario*, the *clean air* scenario, the *bridge* scenario, the *4-for-2* scenario, the *emissions of air pollutants* or the *efficient world* scenario, etc.

**Deep decarbonization pathways** The deep decarbonization pathways project (DDPP)[131] is a global collaboration of energy research teams developing practical decarbonization pathways.

[130] Source: https://www.iea.org/weo/weomodel/.
[131] Source: http://deepdecarbonization.org/about/.

**Energy [r]evolution** Greenpeace developed three scenarios to show the wide range of possible pathways in each world region for a future energy supply system:

- a reference scenario reflecting a continuation of current trends and policies.
- energy [r]evolution scenario: designed to achieve a set of environmental policy targets resulting in an optimistic and feasible pathway towards a widely decarbonized energy system by 2050.
- the advanced energy [r]evolution scenario, representing an ambitious pathway towards a fully decarbonized energy system by 2050.

Other scenarios have been developed, for instance, the international Renewable energy agency (IRENA) proposed the REmap case that leads with a probability of 66% that the increase of temperatures can be kept under 2°C[132]. Broadly, the idea is to propose a sectoral vision of the energy transition pathways in each region.

### A.2.2 IPCC Scenarios

The scenarios discussed by the IPCC are far more detailed plausible representations of the future development of GHG emissions based on a coherent and consistent set of assumptions about driving forces, such as demographic and socioeconomic development, technological change, energy and land use, and their key relationships. In other words, if we previously presented scenario focusing on the energy mix, these scenarios allow more complex interaction between energy side and socioeconomic environment. Each scenario is defined in completion with:

- A shared socio-economic pathway (SSP)[133],
- A representative concentration pathway (RCP)
  These pathways are time series of emissions and concentrations of GHGs. The term pathway emphasizes that not only are the long-term concentration levels of interest, but also the trajectory taken over time to reach that outcome (IPCC; Moss *et al.*, 2010). Four RCPs produced from IAMs selected by the IPCC are used in IPCC Assessment as a basis for the climate predictions (IPCC):
  - *RCP 2.6*:
    *One pathway where radiative forcing peaks at approximately 3 W/ m$^2$ before 2100 and then declines (the corresponding ECP assuming constant emissions after 2100).*
  - *RCP 4.5 and RCP 6.0*:
    *Two intermediate stabilization pathways in which radiative forcing is stabilized at approximately 4.5 W/m$^2$ and 6.0 W/m$^2$ after 2100 (the corresponding ECPs assuming constant concentrations after 2150).*

[132] The Cumulative $CO_2$ by 2050: 760 Gt and Annual CO2 in 2050: 9.7 Gt/yr (Global Energy transformation, IRENA, 2018).
[133] See Section 3.5 on page 53.

– *RCP 8.5*:
*One high pathway for which radiative forcing reaches ≥ 8.5 W/m² by 2100 and continues to rise for some amount of time (the corresponding ECP assuming constant emissions after 2100 and constant concentrations after 2250).*

- The social policy assumptions (SPA)
These parameters are policies that appear to be likely given the political environment in each region and the observed trends. For instance, these assumptions are what distinguish the IEA's NPS and CPS energy scenarios.

## A.3 Portfolio alignment methodologies

The carbon intensity or footprint of the portfolio is often computed and compared to IEA projection to assess the alignment of the portfolio with the 2°C scenario. However, climate alignment and particularly portfolio alignment is still a quite blurry concept. In simple terms, there is no one way to define one's '*alignment*' with respect to what everyone else is doing simultaneously. For instance, it "is defined as the compatibility of projected future production of the companies in the portfolio with energy and technology trends in the 2°C scenario" according to the 2°C investing initiative. This definition leads to assessing the technology mix exposure, therefore the 2°C portfolio must be in line with IEA estimate of the required energetic mix to remain under 2°C by 2100. Other would prefer the sectoral decarbonization approach or GHG emission per value added: defined as the matching of issuers *science based targets*[134] with IEA requirements. The varying definitions lead to some comparative studies of the methodologies. Faria and Labutong[135] (2015), defined three main possible mathematical expressions: "linear reductions from base year to a pre-defined target (LERTY); value added methods (GEVA and C-FACT); and the sectoral decarbonization approach (SDA)". The different methodologies studied have the following expressions:

- *LERTY*
If reduction $\mu_R$ is required, then target year ($\mathcal{E}_{ty}$) emissions will be $(1-\mu_R)$ multiplied by emissions in base year ($\mathcal{E}_{by}$):
$$\mathcal{E}_{\text{ty}} = \mathcal{E}_{\text{by}} \times (1-\mu_R) \tag{20}$$
The linear pathway between base year and target year with $T = ty - by$ (number of years to target) and $n = y - by$ (years after base year):
$$\mathcal{E}_n = \mathcal{E}_{\text{by}} \times \frac{\mathcal{E}_{\text{by}} \times \mu_R}{T} \times n$$

- *C-FACT*
Equation (20) is used to define emissions in target year. The target is given as an intensity

[134] See science based targets in the glossary.
[135] Download available here: https://www.researchgate.net/profile/Pedro_Faria2/publication/275210159_A_Review_of_Climate_Science_Based_GHG_Target_Setting_Methodologies_for_Companies/links/553504ac0cf2ea51c1338d55

in $tCO_2/\$$ following:

$$\mathrm{CI}_n = \mathrm{CI}_{\mathrm{by}} \times \left( \frac{(1-\mu_R)^{1/T}}{(1+\mathrm{CAGR})} \right)^n \tag{21}$$

where $(CI_{by})$ is the initial base year emissions and the gross profit of company in the base year which determine the company's base year intensity; the emission reduction imposed by the model for target year $\mu_R$ that depends on geography[136]; the compound annual growth rate (CAGR), used to calculate the projections into the future of the company's gross profits. The term $(1-R)^{(1/T)}$ Equations (21) and (22) gives the CEDR (Compound Emission Decarbonization Rate).

$$\mathcal{E}_n = \mathcal{E}_{\mathrm{by}} \times (1-\mu_R)^{1/T} \tag{22}$$

- *GEVA*
  The arithmetic approximation (and simpler) version of geometric Equation (21) follows:

$$\mathrm{CI}_n \quad = \quad \mathrm{CI}_{by} \times \left( (1-\mu_R)^{1/T} - \mathrm{CAGR} \right)^n \tag{23}$$

  Likewise, Equation (24) results from applying the same simplifications to (23) to derive $E_n$.

$$\mathcal{E}_n \approx \mathcal{E}_{by} \times (1-\mu_R)^{1/T} \tag{24}$$

  Despite this apparent difference in formulation, the main difference between GEVA and C-FACT is that all companies are assumed to grow at the same rate as the projected world economy and all companies have to equally reduce emissions by 50% by 2050 (outdated targets).

- *sectoral decarbonization approach (SDA)*
  For homogeneous sectors, the emissions in the target year will be equal to the company intensity times the activity in the target year:

$$\mathcal{E}_{ty} = \mathrm{CI}_{\mathrm{ty}} \times A_{ty}$$

  where the parameters can vary from one company to another. Company intensity is determined by:

$$\mathrm{CI}_y = d \times p_y \times m_y + \mathrm{SI}_{2050}$$

  which sets a convergence pathway from the company intensity in the base year to the sector intensity required in 2050 ($SI_{2050}$). The parameters of the expression are $d$ the distance between company intensity in the base year and the sector intensity in 2050:

$$d = \mathrm{CI}_b - \mathrm{SI}_{2050} \tag{25}$$

[136]For instance, the reduction target were 85% of GHG in developed countries and 50% in developing countries in 2015 (Faria and Labutong, 2015).

$p_y$ which gives the pace at which the convergence will occur. It follows the shape of the sector intensity pathway

$$p_y = \frac{\mathrm{SI}_y - \mathrm{SI}_{2050}}{\mathrm{CI}_b - \mathrm{SI}_{2050}}$$

$m_y$ is a corrective factor that modulates the company intensity depending whether the company grows faster or slower than the sector activity:

$$m_y = \frac{\mathrm{CA}_b/\mathrm{SA}_b}{\mathrm{CA}_y/\mathrm{SA}_y}$$

For heterogeneous sectors, the value-added method is used and so expressions are formally equivalent to the ones presented in C-FACT and GEVA. The main difference is that all assumptions concerning growth of the sector and its reductions are concentrated in the sector $CO_2$ emissions pathways derived from IEA modeling. Thus, the reduction $\mu_R$ to target year is given by $(\mathrm{SE}_{\mathrm{ty}}/\mathrm{SE}_{\mathrm{by}})$ and the company intensity for a given year $y$ is simply given by $(\mathrm{SE}_y/\mathrm{SE}_{\mathrm{by}})$.

$$\mathcal{E}_{\mathrm{ty}} = \mathcal{E}_{\mathrm{by}} \times (1 - \mu_R) \tag{26}$$

$$\mathrm{CI}_y = \mathrm{CI}_{\mathrm{by}} \times (\mathrm{SE}_{\mathrm{ty}}/\mathrm{SE}_{\mathrm{by}}) \tag{27}$$

This study also shows that the methods are varyingly effective according to the sector and that the correct / optimal answer could be based on their optimal combination. The targets in this presentation are outdated and must be adjusted to each regulatory announcement. Here again, the weighted sum of carbon intensities can lead to the construction of 'aligned portfolios'.

These varying definitions, based on varying requirements and projections raise the question of feasibility and concrete implementation of this concept in multi-asset allocation strategies. Issuers' mitigation strategies with respect to the global expected reduction of GHG emissions must be defined in order to allow allocations that reduce the risk of divergence (from the 2°C reference). The challenge is then to assess the dynamic evolution of issuers' mitigation over the long-term. Another definition we could think of could be based, for example, on the cap-weighted amount $I_{\mathrm{trans}}$ of securities favoring the transition in the portfolio; requiring it to be equal to the optimal Pigouvian tax, and balancing the SCC.

## A.4 P9XCA: Accounting for induced emissions

P9XCA is an accounting methodology resulting from the partnership between the chair of Finance and Sustainable Development (Finance et Developement Durable) Paris Dauphine University (P9) Ecole Polytechnique (X) and Crédit Agricole (CA) and presented in Rose's (2014) thesis. This methodology aims at defining a sectoral and regional mapping of the order of magnitude of GHG induced by financial activities (Rose, 2014). This methodology is said to be applicable to any financial activity without multiple counting using the accounting by issue approach. The main principles are:

- A macro-economic approach (top-down) adapted to a banking activities portfolio.
- Simple methodology to define the induced emissions $I_e(s,c,i)$, by the financial actor $i$ for the sector $s$ in the country $c$ based on the following equations:

$$
\begin{aligned}
I_e(i) &= \sum_{k\in c}\sum_{p\in s} O_a(p,k,i) \times \mathbb{A}_{p,k} && (28)\\
I_e(s,c,i) &= O_a(s,c,i) \times \mathbb{A}_{s,c} && (29)\\
I_e(s,c,i) &= O_a(s,c,i) \times \frac{\mathcal{E}_{\text{tot}}(s,c)}{(\text{Equity}+\text{Dept})(s,c)} && (30)\\
I_e(s,c,i) &= S_{f,i} \times E_{\text{tot}}(s,c) && (31)
\end{aligned}
$$

  where $O_a(s,c,i)$ is the outstanding amount of the financial actor $i$ in the cluster $(s,c)$, $\mathbb{A}_{s,c}$ is the emission intensity factor of this cluster, $\mathcal{E}_{\text{tot}}(s,c)$ is its global effective annual emissions flow. The financial share $S_{f,i}$ is defined as the fraction of banking outstanding amount in the cluster $(s,c)$ with the sum of both equity and debt volumes. This allows to assess the risk of multi-asset strategy (not only equity).

- No multiple counting because of the use of account-by-issue technique[137].
- Based on public, free and open-source data (National Reporting Inventories), to avoid to relying on black box private processes.

This model is recognized to be mostly a reporting tool, not precise enough to drive, or control, the allocation decision process. However, it allows us to simply assess the emissions induced by a portfolio (if the mapping between issuers and sectors is reliable).

## A.5 MPC–DICE Optimization

This section provides a review of the formalized optimal control problem proposed by Kellett *et al.* (2018). The discrete indexation is defined such as: $\Delta t = 5$ years. It will be noted $\Delta$ for simplicity in accordance with the notation used in the original paper. We have previously introduced the six endogenous variables of the model: 2 temperatures (in °C): $\mathcal{T}_{\text{AT}}$, $\mathcal{T}_{\text{LO}}$; 3 carbon concentrations (in $\text{GtCO}_{2eq}$): $\mathcal{C}_{\text{AT}}$, $\mathcal{C}_{\text{UP}}$, $\mathcal{C}_{\text{LO}}$, and the global capital (in trillionsof 2005USD): K. They answer the following dynamics:

$$
\begin{aligned}
\begin{bmatrix} \mathcal{T}_{\text{AT}}(t+1) \\ \mathcal{T}_{\text{LO}}(t+1) \end{bmatrix} &= \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} \mathcal{T}_{\text{AT}}(i) \\ \mathcal{T}_{LO}(i) \end{bmatrix} + \begin{bmatrix} \xi_1 \\ 0 \end{bmatrix} \mathcal{F}_{\text{RAD}}(t) && (32)\\
\begin{bmatrix} \mathcal{C}_{\text{AT}}(t+1) \\ \mathcal{C}_{\text{UP}}(t+1) \\ \mathcal{C}_{\text{LO}}(t+1) \end{bmatrix} &= \begin{bmatrix} \xi_{11} & \xi_{12} & 0 \\ \xi_{21} & \xi_{22} & \xi_{32} \\ 0 & \xi_{32} & \xi_{33} \end{bmatrix} \begin{bmatrix} \mathcal{C}_{\text{AT}}(t) \\ \mathcal{C}_{\text{UP}}(t) \\ \mathcal{C}_{\text{LO}}(t) \end{bmatrix} + \begin{bmatrix} \xi_2 \\ 0 \\ 0 \end{bmatrix} \mathcal{E}(t) && (33)
\end{aligned}
$$

137 See account-by-issue in the glossary.

$$
\begin{aligned}
K(t+1) &= (1-\delta_K)^{\Delta} K(t) + \Delta I(t) \\
&= (1-\delta_K)^{\Delta} K(t) + \Delta Q(t) s(t) \\
K(t+1) &= (1-\delta_K)^{\Delta} K(t) + \Delta \left( \frac{1}{1+\theta_2 \mathcal{T}_{AT}(i)^2} \right) (1 - b_1(t)\mu(t)^{b_2}) A_{\mathrm{TFP}}(t) \times \\
& K(t)^{\alpha} L(t)^{1-\alpha} s(t) \qquad (34)
\end{aligned}
$$

were $s(i) = I(i)/Q(i)$ is the saving rate and $Q(t)$ was defined in the previous section using the Cobb-Douglass relationship. The exogenous influential signals are defined by the following relationships:

$$
\begin{aligned}
\mathcal{E}(t) &= (1-\mu(t))\, \sigma(t) A_{\mathrm{TFP}}(t) K(t)^{\alpha} L(t)^{1-\alpha} + E_{\mathrm{land}}(t) \\
\mathcal{F}_{\mathrm{RAD}}(t) &= \eta \log_2 \left( \frac{M_{\mathrm{AT}}(t)}{M_{AT}(1750)} \right) + \mathcal{F}_{\mathrm{EX}}(t) \\
\sigma(t+1) &= \sigma(t) \exp(-g_{\sigma}(1-\delta_{\sigma})^{\Delta(t-1)} \Delta) \\
L(t+1) &= L(t) \left( \frac{L_{\infty}}{L(t)} \right)^{l_g} \\
A_{\mathrm{TFP}}(t+1) &= \frac{A_{\mathrm{TFP}}(t)}{1 - g_A \exp(\delta_A \Delta (t-1))} \\
\mathcal{E}_{\mathrm{land}}(t) &= E_{\mathrm{LO}} (1-\delta_{\mathrm{EL}})^{t-1} \\
\mathcal{F}_{\mathrm{EX}}(t) &= f_0 + min \left\{ f_1 - f_0, \frac{f_1 - f_0}{t_f}(t-1) \right\} \\
\theta_1(t) &= \frac{p_b}{\theta_2} (1-\delta_{pb})^{t-1} \sigma(t)
\end{aligned}
$$

were the parameters are given Table 3.The utility function represented by the social welfare, follows:

$$
u\left(C(t), L(t)\right) = L(t) \times \frac{\left(\frac{C(t)}{L(t)}\right)^{1-\theta} - 1}{1-\theta}
$$

and the consumption ($C$) is:

$$
C(i) = \frac{1}{1 + a_2 \mathcal{T}_{AT}^{a_3}} (1 - \theta_1 \mu(t)^{\theta_2}) A_{\mathrm{TFP}}(t) K(t)^{\gamma} L(t)^{1-\gamma} (1 - s(t))
$$

The optimal pathways are derived by maximizing the social welfare at each step (infinite horizon optimal control model):

$$
\begin{aligned}
\max_{s,\mu} \Delta \times scale_1 \times \quad & \sum_{t}^{\infty} \frac{U[C(t), L(t)]}{(1+\rho)^{\Delta}} - scale_2 \qquad (35) \\
\text{subject to} \quad & (32) - (34) \\
& \mu(t), s(t) \in [0,1]^2 \quad \forall t \in [1, T]
\end{aligned}
$$

The social cost of carbon is defined by the ratio of the marginal welfare with respect to emissions and consumption:

$$
SCC(t) = \times \frac{\partial W / \partial \mathcal{E}(t)}{\partial W / \partial C(t)}
$$

The state vector is defined as follows:

$$\begin{aligned}\tilde{x} &= [t\ T\ \mathcal{C}\ K\ \sigma\ L\ A_{\text{TFP}}\ \mathcal{E}_{\text{land}}\ \mathcal{F}_{\text{EX}}]^\top \\ x_{\text{aux}} &= [\mathcal{E}(t)\ C(t)\ \mu(t)\ s(t)\ W(t)]^\top\end{aligned}$$

$\tilde{x}$ embeds 12 dimensions, $x_{\text{aux}}$ describes the behavior of the five variables above. Combining the two we obtain:

$$\begin{aligned}x(t) &= [\tilde{x}(t)^\top\ x_{\text{aux}}(t)^\top]^\top \\ w(t) &= [\mu(t+1)\ s(t+1)]^\top \\ x(t+1) &= f(x(t), w(t)), \quad x(1) = v\end{aligned}$$

where $f : \mathbb{R}^{17} \times \mathbb{R}^2 \to \mathbb{R}^{17}$ is given by (10) to (35). This function basically translates the system dynamics. For instance, $f_1$ is the indexation, $f_2$ to $f_4$ translate temperature dynamics and so on. $f_{17}$ is the update of the social welfare utility function such as:

$$x_{17}(t+1) = x_{17}(t) + \frac{U(x_{12}(t), x_9(t))}{(1+\rho)^{\Delta(t-1)}} \quad x_{17}(1) = 0$$

Algorithm 1 returns (Kellett *et al.*, 2018):

- The *optimal state trajectory* $x^*(t)$; $t = 1, ..., N+1$, which contains the savings rate, $x^*_{15}(t)$, and the mitigation rate, $x^*_{16}(t)$.
- The *optimal adjoint variables* $\lambda^*_E$ and $\lambda^*_C$ which are given by the Lagrange multipliers associated with the equality constraints implied by the dynamics of $\mathcal{E}(t) = x_{13}(t)$ and $C(t) = x_{14}(t)$.

The Lagrange multipliers are typically provided by modern nonlinear programing solvers such as interior-point optimizer (IPOPT; Wächter, 2006).

**Algorithm 1:** Model Predictive Control – Dynamic Integrated Climate Economy – Proposed by Kellett *et al.* (2018)

**Input:**
Simulation horizon $N_{sim}$,
Prediction horizon $N$,
Initial conditions $x(1) = \upsilon$
Dice dymamics function $f : \mathbb{R}^{17} \times \mathbb{R}^2 \to \mathbb{R}^{17}$
**for** $i == 1$ **do**

$$\max_{w,\upsilon} x_{17}(N+1)$$

$$\begin{aligned}
&\text{subject to:} \\
x(i+1) &= f(x(i), w(j)) \\
x(1) &= \upsilon \\
\upsilon_k &= x_k(1) \quad k \in \{1, ...17\} \setminus \{15, 16\} \\
\upsilon_k &\in [0,1] \quad k = 15, 16 \\
w(i) &\in [0,1]^2 \quad \forall i \in [1, N]
\end{aligned}$$

**Set**

$$\begin{aligned}
x(1) &\leftarrow x^*(1|1) \\
\lambda_E(1) &\leftarrow \lambda_E^*(1|1) \\
\lambda_C(1) &\leftarrow \lambda_C^*(1|1)
\end{aligned}$$

**for** $i == 2, ..., N_{sim}$ **do**

$$\max_{w,\upsilon} x_{17}(N+1)$$

$$\begin{aligned}
&\text{subject to:} \\
x(i+1) &= f(x(i), w(j)) \\
x(1) &= x^*(2|i-1) \\
w(i) &\in [0,1]^2 \quad \forall i \in [1, N]
\end{aligned}$$

**Set**

$$\begin{aligned}
x(i) &\leftarrow x^*(2|i-1) \\
\lambda_E(i) &\leftarrow \lambda_E^*(2|i-1) \\
\lambda_C(i) &\leftarrow \lambda_C^*(2|i-1)
\end{aligned}$$

**Result:** Optimal trajectory for state variable $x^*(i)$ $\forall i \in [1, N_{sim}]$

## A.6 Complementary materials

Figure 15: DICE 2013 - 2°C

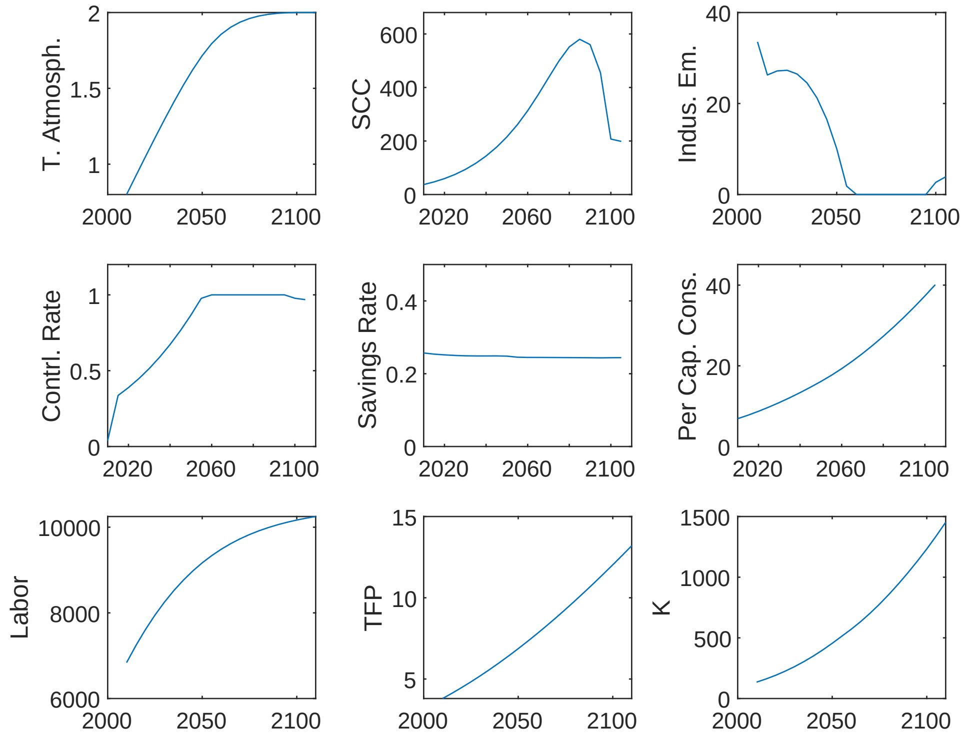


Table 6: NACE Rev.2 sectors (level 1)

| Sector code | Sector description |
|---|---|
| A | Agriculture, forestry and fishing |
| B | Mining and quarrying |
| C | Manufacturing |
| D | Electricity, gas, steam and air conditioning |
| E | Water supply, sewerage, waste management and remediation activities |
| F | Constructions |
| G | Wholesale and retail trade, repair of motor vehicles and motorcycles |
| H | Transportation and storage |
| I | Accommodation and food service activities |
| J | Information and communication |
| K | Financial and insurance activities |
| L | Real estate activities |
| M | Professional, scientific and technical activities |
| N | Administrative and support service activities |
| O | Public administration and defense, compulsory social security |
| P | Education |
| Q | Human health and social work activities |
| R | Arts, entertainment and recreation |
| S | Other services activities |
| T | Activities of households as employers, undifferentiated goods and service |
| U | Activities of extraterritorial organizations ans bodies |

Source: https://ec.europa.eu/eurostat/ramon/nomenclatures/index.cfm?TargetUrl=LST_NOM_DTL&StrNom=NACE_REV2&StrLanguageCode=EN.

Figure 16: DICE 2013–Optimal Control Rate

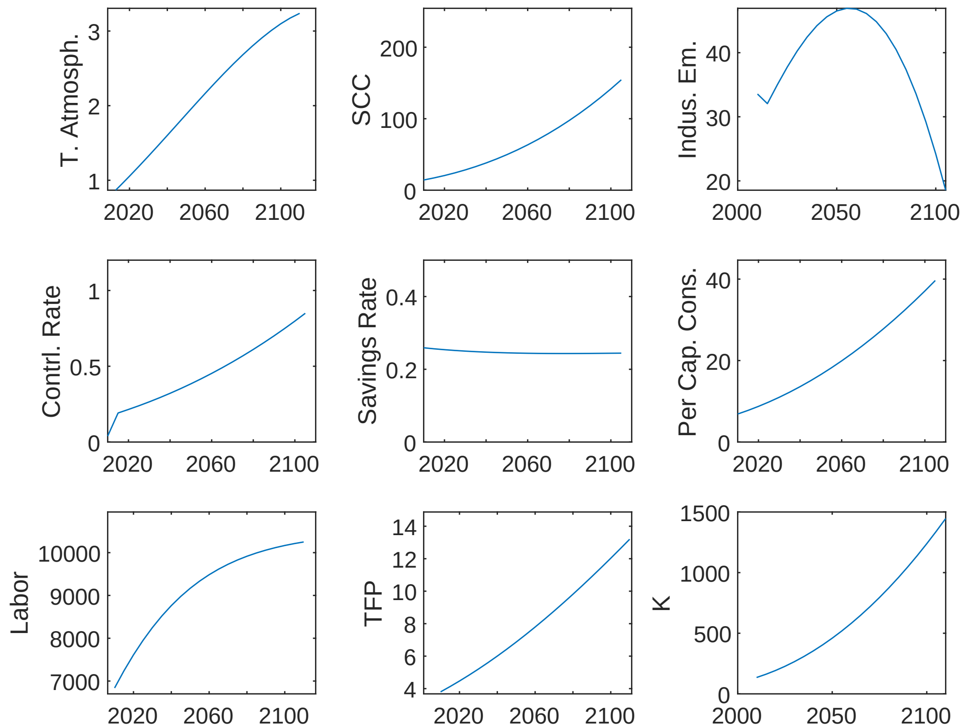
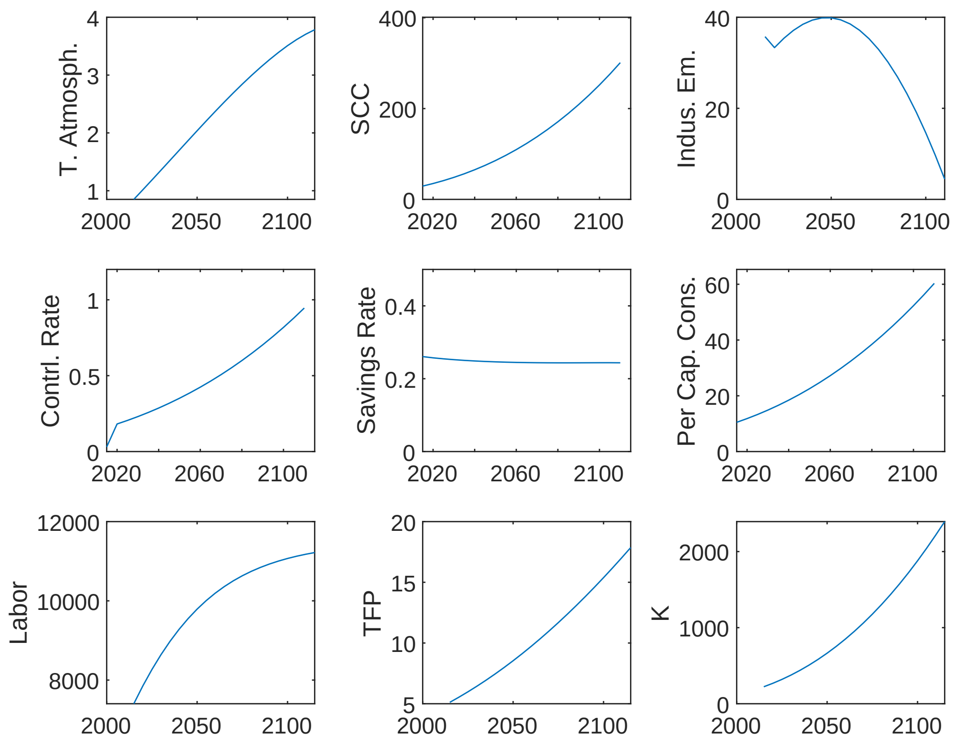



Figure 17: DICE 2016–Optimal Control Rate



# B Glossary

**Abatement Costs** also called mitigation costs, they are the costs of reducing environmental negatives. In practice this cost are given in term of relative production, we talk therefore about marginal cost, which measures the cost of reducing by an additional unit. The marginal abatement cost (MAC), measures consequently the costs of reducing GHG emissions by one additional unit and generally measured in $\$/tCO_{2eq}$. They are usually deterministic static function with poor representation of spacial and temporal dynamics. Sectors specificities are poorly represented despite the increasing complexity of the models. These functions have therefore been largely criticized by the academic literature (Kesicki and Ekins, 2012; Levihn, 2016; Taylor, 2012; Ward, 2016; Wallis, 1992). They are often computed as "proportional to global output and to a polynomial function of the reduction rate", $\mu(t)$. the parameters are usually fitted on bottom-up studies such as McKinsey (2009) estimates for Daniel *et al.* (2018) (similarly as Nordhaus):

$$\Lambda(\mu(t)) = b_1 \mu(t)^{b2}$$

For a more complex example, the RESPONSE (Dumas *et al.*, 2012) model proposed the following abatement cost function:

$$\begin{aligned}\Lambda(\mu(t)) &= A_{\mathrm{TFP}}(\mu(t)\zeta + (p_{\mathrm{B}} - \zeta)\frac{(\mu(t))^\nu}{\nu} + \\ &\xi^2(\mu(t) - \mu(t-1))^2)\end{aligned}$$

were:

- $\mu(t)$ mitigation rate at t,
- $p_{\mathrm{B}}$ price of the backstop technology,
- $\xi$ inertia effect (Ha-Duong *et al.*, 1997), introducing a penalization when the increase of the reduction is too abrupt,
- $\zeta$ is the marginal cost of abatement when abatement is null,
- $A_{\mathrm{TFP}}$ represents the technical changes factor.

Other model representing abatement cost can be developed (see for example Kiuila and Rutherford, 2011)[138] but the questions about their lack transparency and the poor treatment it makes of uncertainty (etc.) remains.

**Account-by-Issue** is defined by Rose (2014) as follow: "the economic issue or challenge of an economic agent is the amount of GHG emissions that this agent would be likely to reduce in an economy where strong constraints would be introduced on GHG emissions. These constraints on GHG emissions translate into carbon cost, the internalization of these additional costs leads to a decrease in demand for carbon goods in favor of less carbonaceous goods."[139] The central idea is that this type of carbon accounting allocates GHG emmissions oberved to economic agents possessing the decision making and leverage power to reduce them. Accountability carbon by stake makes it possible to explain the "fundamental responsibility " of each activity economic, which corresponds to the choice of the technological processes that it implements in its activity, and the quantity and quality of the goods and services it offers[140] (Rose, 2014). This accounting method particularly allow to avoid multiple counting, while accounting based on carbon scopes (see Carbon Scopes) can not avoid this.

**AOGCMs** for *Atmosphere-Ocean Global Circulation Models* are climate models, using Navier-Stokes equation on rotating sphere and thermodynamics terms (radiation, latent heat), that simulate Atmosphere and Ocean behavior. It can be of use to *dispatch* geographically the consequences of a rising temperature (see Collins *et al.* 2006; Meinshausen *et al.*, 2011).

**Backstop Technology** "is defined as a new technology producing a close substitute to an exhaustible resource by using relatively abundant production inputs

[138] Download available here: https://www.wne.uw.edu.pl/files/7013/9628/6009/WNE_WP52_2011.pdf

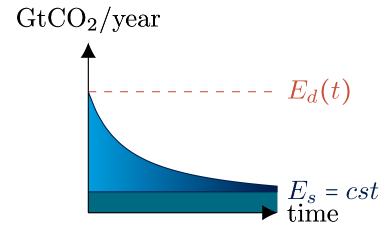
[139] Translated from French: "*l'enjeu climatique d'un agent économique est la quantité d'émissions de GES que cet agent serait susceptible de réduire dans une économie où seraient introduites des contraintes fortes sur les émissions de GES. Sous des contraintes portant sur les émissions de GES qui se traduisent par un coût des émissions, l'internalisation de ces coûts supplémentaires entraı̂ne une diminution de la demande en biens carbonés au profit de biens moins carbonés*" (Rose, 2014).

[140] Translated from French: "*Ce type de comptabilité carbone alloue les émissions de GES observées aux agents économiques possédant le pouvoir de décision et le levier d'action nécessaire pour les réduire. La comptabilité carbone par enjeu permet d'expliciter la* responsabilité fondamentale *de chaque activité économique, qui correspond au choix des procédés technologiques qu'il met en oeuvre dans son activité, et des quantités et de la qualité des biens et services qu'il offre*" (Rose, 2014).

and rendering the reserves of the exhaustible resource obsolete when the average cost of production of the close substitute falls below the spot price of the exhaustible resource (Dasgupta and Heal, 1978). For instance, the technology of harnessing solar energy can be perceived as a backstop technology to oil, coal and natural gas. Hence, the development of a backstop technology shortens the planning horizon and", in turn, "the presence of a backstop technology also lowers the spot prices of the exhaustible resource and accelerates its extraction and depletion" (Levy, 2000).

**Baseline** The baseline (or reference) is the state against which change is measured. A baseline period is the period relative to which anomalies are computed. In the context of transformation pathways, the term baseline scenarios refers to scenarios that are based on the assumption that no mitigation policies or measures will be implemented beyond those that are already in force and/or are legislated or planned to be adopted. Baseline scenarios are not intended to be predictions of the future, but rather counterfactual constructions that can serve to highlight the level of emissions that would occur without further policy effort. Typically, baseline scenarios are then compared to mitigation scenarios that are constructed to meet different goals for GHG emissions, atmospheric concentrations or temperature change. The term baseline scenario is used interchangeably with reference scenario and no policy scenario. In much of the literature the term is also synonymous with the term business-as-usual (BAU) scenario, although the term BAU has fallen out of favour because the idea of business as usual in century-long socio-economic projections is hard to fathom. See also Emission scenario, Representative Concentration Pathways (RCPs) and SRES scenarios (IPCC, 2014).

**Carbon Budget** defines the amount of carbon dioxide that a country, company, or organization has agreed is the largest it will produce in a particular period of time: The Committee on Climate Change will advise the government on staying within its carbon budget (Cambridge dictionary). This concept previously introduced aims to maintain the flow of GHG emissions under a certain level ($E_s$ on the figure below: Carbon Budget and abatement, Source: Reproduced from Vogt-Schilb *et al.*, 2013), the one the environment is able to absorb through oceans, which only shift the problem as it increase their pH (Caldeira and Wickett, 2003), or forests for example. For instance, $\mu_i(t)$ being the abatement realized by the sector $i$ at $t$, we can graphically induce that $B(t) = \int_0^t \sum_i \mu_i(t)\,\mathrm{d}t - tE_s(t)$.




**Carbon Equivalent ($CO_2$eq)** "Carbon dioxide equivalent or CO2e is a term for describing different GHG in a common unit. For any quantity and type of GHG, CO2e signifies the amount of CO2 which would have the equivalent global warming impact. A quantity of GHG can be expressed as CO2e by multiplying the amount of the GHG by its Global Warming Potential (GWP). E.g. if 1kg of methane is emitted, this can be expressed as 25kg of CO2e (1kg CH4 × 25 = 25kg CO2e). CO2e is a very useful term for a number of reasons: it allows bundles of GHG to be expressed as a single number, and it allows different bundles of GHGs to be easily compared (in It is also worth noting that CO2e is also sometimes written as CO2eq, CO2equivalent, or even CDE, and these terms can be used interchangeably"(Brander, 2012).

**Carbon Dioxide Capture and Storage (CCS)** A process in which a relatively pure stream of carbon dioxide from industrial and energy-related sources is separated (captured), conditioned, compressed and transported to a storage location for longterm isolation from the atmosphere (IPCC, 2014).

**Carbon Dioxide Removal (CDR)** Carbon Dioxide Removal methods refer to a set of techniques that aim to remove CO2 directly from the atmosphere by either (1) increasing natural sinks for carbon or (2) using chemical engineering to remove the CO2, with the intent of reducing the atmospheric CO2 concentration. CDR methods involve the ocean, land and technical systems, including such methods as iron fertilization, large-scale afforestation and direct capture of CO2 from the atmosphere using engineered chemical means. Some CDR methods fall under the category of geoengineering, though this may not be the case for others, with the distinction being based on the magnitude, scale and impact of the particular CDR activities. The boundary between CDR and mitigation is not clear and there could be some overlap between the two given current definitions (IPCC, 2012b, p. 2). See also Solar Radiation Management (SRM) (IPCC, 2014).

**Carbon Credits or Licenses** are an allowance that certain companies have, permitting them to burn a certain amount of fossil fuels (Collins dictionary). For a better understanding of this concept go to Emission Trading System.

**Carbon Intensity** is the amount of emissions of carbon dioxide (CO2) released per unit of another variable such as Gross Domestic Product (GDP), output energy use or transport (IPCC, 2014).

**Carbon Footprint** is a measure of the total amount of carbon dioxide (CO2) and methane (CH4) emissions of a defined population, system or activity, considering all relevant sources, sinks and storage within the spatial and temporal boundary of the population, system or activity of interest. Calculated as carbon dioxide equivalent using the relevant 100-year global warming potential (GWP100) (Wright *et al.*, 2011). This metric can be expressed as the absolute value of $tCO_{2eq}/\$$ (or €) invested or as a carbon intensity (used in value added models).

**Carbon Pricing** is the basic idea of putting a price on GHG emissions. The ways to do so are mainly the tax and ETS described in (Carbon Risks, Carbon Credits, and Emission Trading Systems).

**Carbon Risks** gather the potential financial losses, direct or indirect through the value chain, due to GHG emissions. Concretely, it is the financial exposure to reputational risks (economic and market risk) or regulatory (credit and market risks). The regulatory risk can be measured as a conditional Value-at-Risk (VaR) subject to the optimal tax or quotas required to reach, in the hypothesis of a lump-sum refunded carbon tax, to reach, for example the 1.5°C scenario which lead to *zero emission in 2050* (IPCC, 2014; Hulme, 2016).

**Carbon Scopes (ISO 14064)** divides companies' emissions into 3 Scopes according to the Greenhouse Gas Protocol. define the accounting and reporting standards companies can adopt disclosing their emissions.

- *Scope 1: Direct GHG emissions* covers all direct GHG emissions by a company.
- *Scope 2: Electricity indirect GHG emissions* from consumption of purchased power: electricity, heat or steam (etc.). The proper way to quantify this scope would require a suitable mapping of energy mix supply of issuers' facilities. Some have introduced real-time accounting methods based on open sourced data[141] which could allow to better identify issuers contribution to global emission (Tranberg *et al.*, 2018).
- *Scope 3: Other indirect GHG emissions* such as the extraction and production of purchased materials and fuels, transport-related activities in vehicles not owned or controlled by the reporting entity, electricity-related activities (e.g. T&D losses) not covered in Scope 2, outsourced activities, waste disposal, etc. Scope 3 emissions (also known as value chain emissions) often represent the largest source of GHG emissions and in some cases can account for up to 90% of the total carbon impact. To some extent this scope is the main focus of P9XCA method. Indeed, if we focus on scope 1 and 2 the emission induced by banking activity would be negligible.

**Climate Risks** are the risks related to climate sensitivity of a security because of its dependencies or its geographic positions. Therefore, it regroups technological transition and physical risks. This transition concerns the product and not the process. For instance, climate risks for a car manufacturer embeds to what extent the actor is able to shift its business toward a cleaner transportation, but not how much it has to pollute to produce his new products. The second would go into carbon risks. This distinction, made in this paper, is not a commonly accepted reference, however, it allows to better project and understand the stake at an issuer level. More concretely, climate risks embed future expected impact on companies' businesses while carbon risks are mostly quantified by the amount they would have to pay if the optimal Pigouvian tax were implemented (see Pigouvian Tax). We can note that these risks are not totally independent as their prevention results in quite similar concrete actions from companies or policy maker: the first can reduce their GHG emissions and the second can implement policies forcing everyone to do so. It is however more a question of where we stand, and from a portfolio management perspective these two risks must be distinguished in order to by quantified. However, the computation of the optimal tax is based on the impact of emission on future economic well-being and so incorporates damage functions assessing climate risks.

**Cumulative Radiative Force (CRF)** The strength of drivers is quantified as Radiative Forcing in units watts per square meter (W/m2) as in previous IPCC assessments. Radiative Forcing is the change

[141] https://www.electricitymap.org/.

in energy flux caused by a driver and is calculated at the tropopause or at the top of the atmosphere (IPCC, 2014).

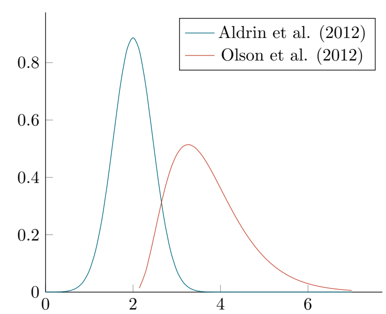
**Discounting and Discount Rates** mathematical operation making monetary (or other) amounts received or expended at different times (years) comparable across time. The discounter uses a fixed or possibly time-varying discount rate (> 0) from year to year that makes future value worth less today (IPCC, 2014). Lower discount rates increase the importance of the outcomes in later years (ex: Stern, 2007). There is no consensus on its value in the literature however Stern value was said to be an outlier.

**Elasticity of Inter-temporal Substitution (EIS)** "One of the important determinants of the response of saving and consumption to the real interest rate is the elasticity of inter-temporal substitution. That elasticity can be measured by the response of the rate of change of consumption to changes in the expected real interest rate" (Hall, 1988). Most IAMs are based on a value of inter-temporal substitution rate.

**Emissions Trading Systems (ETS)** Montgomery (1972) developed a model to define for each firm a single valued function which associate a cost with an emission rate. His modeling was based on the following ideas:

- $n$ agents emitting $e_i$ for a vector of output $Y = (y_{i1}, ..., y_{iR})$, calibrated on optimal inputs;
- A cost function $G_i(Y, e_i)$. The cost, $F_i(e_i)$, of adopting the emission $e_i$ is defined as the difference between its unconstrained maximum of profit and its profits when emission equal $e_i$;
- $m$ location suffering the consequences of the pollution $q_i$, which is defined as a linear combination of the emissions. The relationship between $e$ and $q$ is given by a diffusion matrix $H$, such that: $q = H \cdot e$;
- To define the efficient emission vector, the problem is to choose, $E = (e_1, ..., e_n)$ minimizing $\sum_i F_i(e_i)$ subject to $E \geqslant 0$ and $EH \leqslant Q^*$;
- Then the market is constructed introducing licenses (see Carbon Credit), such that the possession of licenses confers the right to carry a certain average rate of emission, $L_i = (l_{i1}, ..., l_{ik})$.

**Equilibrium Climate Sensitivity (ECS)** Uncertainty is not a concept restricted to economical variables. Our short knowledge of the climate and Atmosphere-Ocean Global Circulation currents leaves space to a range of uncertainty concerning climate parameters that are not known. The most important parameter is the *Equilibrium Climate Sensitivity* (ECS). This parameter represents the equilibrium change in global mean near-surface atmospheric temperature that would result from a sustained doubling of the $CO_2eq$ atmospheric concentration. Olson *et al.* (2012) set the interval [1.1,11.2] (in °C per carbon equivalent doubling). They used Bayesian approach, Gaussian process emulator and Monte Carlo estimations to infer, similarly to other studies, the most likely distribution of this parameter.




A uniform distribution was plotted as a proxy for Aldrin et al. (2012) curve but most studies demonstrated a rather skewed distribution with long tail for higher climate sensitivity. Oslon et al. (2012) for instance used a product of normal inverse Gaussian distribution (NIG). The equilibria given above represents proxies of the distribution obtained by Olson *et al.* (2012) and Aldrin *et al.* (2012) ($\alpha = 0$).

**Green Bonds (GB)** "*are any type of bond instrument where the proceeds will be exclusively applied to finance or re-finance, in part or in full, new and/or existing eligible Green Projects (see Section 1 Use of Proceeds) and which are aligned with the four core components of the GBP*[142]" (Green Bonds Principles, 2018). These are the currently types of green bonds defined in the GBP:

- *Standard green use of proceeds bond:* a standard debt obligation answering the GBP requirement.
- *Green revenue bond:* a non-recourse-to-the-issuer debt obligation aligned with the GBP in which the credit exposure in the bond is to the pledged cash

[142] The full description is available here: https://www.icmagroup.org/assets/documents/Regulatory/Green-Bonds/June-2018/Green-Bond-Principles---June-2018-140618-WEB.pdf.

flows of the revenue streams, fees, taxes etc., and whose use of proceeds go to related or unrelated Green Project(s).

- *Green project bond:* a project bond for a single or multiple green projects for which the investor has direct exposure to the risk of the projects with or without potential recourse to the issuer, and that is aligned with the GBP.
- *Green securitised bond:* a bond collateralised by one or more specific green projects, including but not limited to covered bonds, ABS, MBS, and other structures; and aligned with the GBP. The source of repayment is the cash flows of the assets.

Green bonds used to be mostly issued by investment bank but the issuer profile is slightly evolving and spreading to other actors[143].

**Emissions Trading Systems (ETS)** are a market-based systems where the total amount of emission is capped and restricted (see Carbon Budget) in order to reduce global GHG emissions. This type of cap and trade program allows a flexible environmental regulation while exposure to regulation risk rely on more uncertainty: "we find that policy uncertainty forces organizations to focus their responses on short-term investments and dealing with that very uncertainty, thereby precluding the development of green capabilities and preventing flexible regulations from achieving their intended policy results" (Teeter and Sandberg, 2017). The system allows economic agent to trade their emission permit or allowances on the market, in the case of GHG emission we often speak of Carbon Credit. The one willing to increase its emission would therefore have to 'buy' an allowance from the ones willing to sell them. The remaining and decreasing budget or 'cap' increases the prices answering the law of supply and demand.

**Indirect Emissions** Emissions that are a consequence of the activities within well-defined boundaries of, for instance, a region, an economic sector, a company or process, but which occur outside the specified boundaries. For example, emissions are described as indirect if they relate to the use of heat but physically arise outside the boundaries of the heat user, or to electricity production but physically arise outside of the boundaries of the power supply sector (IPCC, 2014; WG III).

**Induced Emissions ($I_e$)** by financial activities are the weighted emissions over cross-asset universes arising from the activities financed. The quantification of these are of prior importance for asset managers to assess the impact of their investment strategy on the environment (Rose, 2014). In his thesis Rose defined 3 main objectives: (i) definition of green funds and marketing (through green business risk and opportunity analysis, etc.), (ii) measure of the contribution in the fight against climate change and (iii) carbon risks approximation. The methodological issues "bottom-up vs. top-down" is generally a concern to properly account induced emissions.

**Intergovernmental Panel on Climate Change (IPCC)** is an intergovernmental scientific panel of the United Nations committed to develop global projection concerning climate change, and its social, political and economic impacts.

**Keynesian Economics** are theories that emphasis the influence of aggregated demand in the short-run. The main direction these theories gives is that monetary and fiscal policy action by government or central banks can stimulate economic activity and stabilize output over the business cycle.

**Knowledge Factor** This factor reflects the technical progress. The way the knowledge factor $A(t)$ is introduced matters. Moreover, the evolution of this parameter significantly affects climate integrated economic modeling. In Equation (1) on page 9 we considered the progress to be Harrod-neutral meaning that it affects the effectiveness of labor. This factor could also enter in the form $Y(t) = F(A(t)K(t), L(t))$, called capital augmenting or $Y(t) = A(t)F(K(t), L(t))$ where the process is said to be Hicks-neutral, which is adapted to describe an invention which raise the marginal productivity of labor and capital in same proportion. In this form, the progress is exogenous to the economic system. Regardless of how the technical progress is introduced, there will be criticism and endogeneity issues and the academic literature has commented extensively on the effect of this parameter. For instance, some have criticized the later insertion of the productivity factor (exogenous case):

> "*Technological progress is intimately dependent on economic phenomena. The evidence suggests that society may indeed affect the allocation of inventive resources through the market mechanism somewhat as it affects the allocation of economic resources generally. If this is true, then*

[143] https://unfccc.int/sites/default/files/resource/GreenBonds.pdf.

*technological progress is not an independent cause of socio-economic change, and an interpretation of history as largely the attempt of mankind to catch up to new technology is a distorted one*" (Schmookler, 1962, page 1).

**Leakage** or (Carbon Leakage) represents the effect of reported emission from a regulated country/sector to country without regulation. It is "a phenomena whereby the reduction in emissions (relative to a baseline) in a jurisdiction/sector associated with the implementation of mitigation policy is offset to some degree by an increase outside the jurisdiction/sector through induced changes in consumption, production, prices, land use and/or trade across the jurisdictions/sectors. Leakage can occur at a number of levels, be it a project, state, province, nation or world region" (IPCC, 2014).

**Marginal Investment Cost (MIC)** "We call Marginal Investment Cost (MIC) the cost of the last unit of investment in low-carbon capital $c_i'(x_{i,t})$" (Vogt-Schilb *et al.*, 2013). The idea is to model the "the economic efforts being oriented towards building and deploying low-carbon capital in a given sector at a given point in time. While one unit of investment at time $t$ in two different sectors produces two similar goods – a unit of low-carbon capital that will save GHG from $t$ onwards – they should not necessarily be valued equally" (Vogt-Schilb *et al.*, 2013). This concept might allow to define a form of *decarbonization premium* to some extent i.e. the sooner we invest on low-carbon capital in the most carbon intensive sectors to more we can expect our investment to be reevaluated.

**Mitigation (of Climate Change)** A human intervention to reduce the sources or enhance the sinks of GHG. This report also assesses human interventions to reduce the sources of other substances which may contribute directly or indirectly to limiting climate change, including, for example, the reduction of particulate matter emissions that can directly alter the radiation balance (e.g., black carbon) or measures that control emissions of carbon monoxide, nitrogen oxides, Volatile Organic Compounds and other pollutants that can alter the concentration of tropospheric ozone which has an indirect effect on the climate. A Mitigation scenario is a plausible description of the future that describes how the (studied) system responds to the implementation of mitigation policies and measures (IPCC, 2014).

**NAMEA** One of the major issue is the unequal policy context in terms of carbon pricing. The main challenge most model fail to take account of if the accountability issues and potential leakage. To give a prior approach of this complexity in a more concrete case, we can rely for on accounting methods such as the *National Accounting Matrix including Environmental Accounts*, which combines Leontief (1970) conventional national input-output accounting approach and environmental physical accounts. It is an environmental accounting framework developed by Statistics Netherlands at the end of the 1980s (De Haan and Keuning, 1996; Keuning *et al.*, 1999). It consists of a conventional national accounting matrix extended with environmental accounts in physical units. They are also called '*hybrid*' accounts. The original idea of Leontief matrices is to describe the economic equilibrium of $n$ products and services, over $n$ sectors. The following equation translate the supply (left side) and demand (right side) equilibrium for $(i,j) \in (n,n)$ (Lenglart *et al.*, 2010):

$$[Y_i] + [Sm_i] + [T_i] + [M_i] = [\mathrm{ID}_{ij}] \cdot 1 + [D_i]$$

For each product or service, the cost are defined by the sum of the production $P_i$, sales margin $Sm_i$ taxes $T_i$ and importation $M_i$, while the demand is defined as the sum of the intermediate consumption $\mathrm{ID}_{ij}$ of the user industries and the final demand $D_i$. The technical coefficients $A_{i,j}$, defines the production of product $i$ required for the production of one unit of product $j$. Then, the production $Y$ and importation $M$ to answer the final demand are defined:

$$\begin{aligned} A_{i,j} &= \frac{\mathrm{ID}_{ij}}{Y_j} \\ Y &= \left(I - A^d\right)^{-1} \cdot D^d \\ M &= A^m \cdot \left(I - A^d\right)^{-1} \cdot D^d + D^m \end{aligned}$$

Based on this model, we will define a matrix calculation process allowing to find any country induced emissions. The following equations are based on the strong assumption that "emission related to the sector j is structurally proportional to the quantity $Y_j$ it produces" (Lenglart *et al.*, 2010), the introduction of the $CO_{2eq}$ emitted on the domestic territory, $E^d = \left(E_j^d\right)$ let us define the $CO_{2eq}^d$ implied by the domestic production. Given the notation of Lenglart *et al.* (2010), $\langle \cdot \rangle$ is the operator transforming a column vector into a diagonal square matrix. We can define the domestic emissions related to the final demand:

$$\begin{aligned} CO_{2eq}^d = \left(CO_{2eq,j}^d\right) &= \left(\frac{E_i^d}{Y_j}\right) = \langle Y \rangle^{-1} \cdot E^d \\ E^{D,d} &= \left\langle D^d \right\rangle \left(I - {}^tA^d\right)^{-1} \cdot CO_{2eq}^d \end{aligned}$$

On the other hand, emissions induced by importations have to be assessed. The same process of accounting can be implemented, which require to model the $CO_{2eq}^{d*}$ matrix representing the overall emission of $CO_{2eq}$ everywhere excluding the domestic territory (Equation (36)). Therefore, for the $c$ exporting countries, we must compute the emission due to the importation (Equation (37)):

$$CO_{2eq}^{d*} = \sum_c \langle \pi_c \rangle \left(I - {}^tA^c\right)^{-1} \cdot CO_{2eq}^c \tag{36}$$

$$E^m = \langle M \rangle \left(I - {}^tA^{d*}\right)^{-1} \cdot CO_{2eq}^{d*} \tag{37}$$

$$\begin{aligned} E^{D,m} &= M^D \left(I - {}^tA^{d*}\right)^{-1} \cdot CO_{2eq}^{d*} \qquad (38) \\ &= \left(\langle D^d \rangle \left(I - {}^tA^d\right)^{-1} \cdot {}^tA^m + \langle D^m \rangle\right) \cdot \\ &\quad CO_{2eq}^{d*} \end{aligned}$$

$\pi_c$ is a column vector which designs the proportion of the importation from the country $c$ in the overall importation of the country considered, which imply that:$\sum_c \langle \pi_c \rangle = I$. The two components (Equation (38)) correspond to emissions related to intermediary consumption, and the ones aimed at answering the final demand.

**Neoclassical Economics** describe individuals maximizing their utility (or profit) given their rational expectation in a market defined by supply and demand law.

**Physical Risks** are environmental risk factors that can be opportunity, uncertainty and hazard-based risks. They are fundamental in the current economic landscape were most are becoming aware of the coming changes. They include chronic weather changes, affecting for instance agriculture or tourism, and potentially more frequent and extreme events and climate disasters: flooding, sea level rise, wildfires, storms, etc. Real estate is also particularly sensitive to the so-called physical risks and investors with assets in coastal areas might for instance, want to assess *their assets at risk flooding*.

**Pigouvian Taxes** "is the generic term for taxes designed to correct inefficiencies of the price system that are due to negative external effects" (Sandmo, 2008). Following, "Pigou (1920), optimal usage of the atmosphere's capacity to absorb GHGs can be obtained, in both theory and practice, when individuals are charged the full social cost of each ton they emit into the atmosphere, or conversely the benefits that accrue to society with the reduction of GHG emissions by one ton. The cost of putting an additional ton of CO2 into the atmosphere at any given time $t$, assuming an optimal emissions reductions pathway in the future, is commonly known as the optimal $CO_2$ price" (Daniel *et al.*, 2018).

**Rational Expectations** Rational expectations in economics is the hypothesis stating that agents' predictions of the future is based on the information available and what they learned from past trends.

**Return to Scale** Economic production functions are defined on the basis of input factors such as labor and capital. For short-term projections, this type of macroeconomic model allows forecasts to be made by extending the time series trends observed for input factors, most commonly using the Cobb-Douglas function. In the long run, these input factors become variable. In thise context, the return to scale is defined as the change in output as factor inputs change in the same proportion. In economics, we use constant returns to scale, which means that the output increases in the same proportion to the increase in all the inputs or factors of production. This is done for the sake of representativeness. This assumption is explained by Romer (2006) as follows:

> "*In a very small economy, there are probably enough possibilities fo further specialization that doubling the amounts of capital and labor more than doubles the output. The Solow model assumes, however, that the economy is sufficiently large that, if capital and labor double, the new inputs are used in essentially the same way as the existing inputs, and thus that output doubles*" (Romer, 2006, page 10).

This condition has major consequences as it affects the convergence of linear programing algorithms and can represent a pitfall when integrating endogenous technical change. Indeed, a nested structure with constant elasticity of substitution production function can lead optimizers to multiple local solutions with questionable global significance:

> "*Linear programming methods, as frequently used in CGE models, are best suited to solving problems with a single maximum. This is usually guaranteed by the adoption of constant or decreasing returns to scale in production functions. The introduction of increasing returns to scale for a part of the model may generate local minima and maxima and has been found in some cases to destabilize the model, such that finding a solution depends critically on the parameter values used*" (Khöler, 2006, page 46).

**Science Based Target (SBT)** Emissions reductions targets adopted by companies to reduce GHG emissions are considered "science-based" if they are in line with the level of decarbonization required to keep global temperature increase below 2°C compared to pre-industrial temperatures, as described by the IPCC[144]. For more information go to: https://sciencebasedtargets.org/methods/. The eligibility criteria gather (reproduced from the website):

- *Boundary:* The target must cover company-wide Scope 1 and Scope 2 emissions and all relevant GHGs as required in the GHG Protocol Corporate Standard.
- *Time frame:* The target must cover a minimum of 5 years and a maximum of 15 years from the date of announcement of the target.
- *Level of ambition:* At a minimum, the target will be consistent with the level of decarbonization required to keep global temperature increase to 2°C compared to pre-industrial temperatures, though we encourage companies to pursue greater efforts towards a 1.5° trajectory.
- *Scope 3:* An ambitious and measurable Scope 3 target with a clear time-frame is required when Scope 3 emissions cover a significant portion (greater than 40% of total scope 1, 2 and 3 emissions) of a company's overall emissions. The target boundary must include the majority of value chain emissions as defined by the GHG Protocol Corporate Value Chain (Scope 3) Accounting and Reporting Standard (e.g. top 3 categories, or 2/3 of total scope 3 emissions).
- *Reporting:* The company will disclose company-wide GHG emissions inventory on an annual basis.

**Social Cost of Carbon (SCC)** designs "a first estimate of the Pigou tax that should be placed on carbon dioxide emissions. Indeed, if the SCC is computed along a trajectory in which the marginal costs of emission reduction equal the SCC, the SCC is the Pigou tax" (Tol, 2008; Carbon Risk; Pigouvian Tax). It can also be defined as the net present value of climate damages (IPCC, 2014). The formal definition in DICE is given on page 27 but broadly correspond to the ratio of standard variations of consumption on emissions.

**Stranded Assets** are assets that have suffered from unanticipated or premature write-downs, devaluations or conversion to liabilities. They can be caused by a range of environment-related risks and these risks are poorly understood and regularly mis-priced, which has resulted in a significant over-exposure to environmentally unsustainable assets throughout our financial and economic systems. Current and emerging risks related to the environment represent a major discontinuity, able to profoundly alter asset values across a wide range of sectors. Some of these risk factors include: (Caldecott *et al.*, 2014).

- Environmental challenges (e.g. climate change, water constraints)
- Changing resource landscapes (e.g. shale gas, phosphate)
- New government regulations (e.g. carbon pricing, air pollution regulation)
- Falling clean technology costs (e.g. solar PV, onshore wind)
- Evolving social norms (e.g. fossil fuel divestment campaign) and consumer behavior (e.g. certification schemes)
- Litigation and changing statutory interpretations (e.g. changes in the application of existing laws and legislation)

**Tipping Point** A level of change in system properties beyond which a system reorganizes, often abruptly, and does not return to the initial state even if the drivers of the change are abated. For the climate system, it refers to a critical threshold when global or regional climate changes from one stable state to another stable state. The tipping point event may be irreversible (IPCC, 2014).

**Turnpike properties** have been established long time ago in finite-dimensional optimal control problems arising in econometry. They refer to the fact that, under quite general assumptions, the optimal solutions of a given optimal control problem settled in large time consist approximately of three pieces, the first and the last of which being transient short-time arcs, and the middle piece being a long-time arc staying exponentially close to the optimal steady-state solution of an associated static optimal control problem (Trelat and Zuazua, 2014).

[144] https://sciencebasedtargets.org/wp-content/uploads/2017/01/EligibilityCriteria.docx.pdf.

**Tragedy of the Commons** "The tragedy of the commons reappears in problems of pollution. Here it is not a question of taking something out of the commons, but of putting something in – sewage, or chemical, radioactive, and heat wastes into water; noxious and dangerous fumes into the air – and distracting and unpleasant advertising signs into the line of sight. The calculations of utility are much the same as before. The rational man finds that his share of the cost of the wastes he discharges into the commons is less than the cost of purifying his wastes before releasing them. Since this is true for everyone, we are locked into a system of 'fouling our own nest' so long as we behave only as independent, rational, free-enterprisers" (Hardin, 1968).

**Vector Autoregression (VAR)** is a stochastic autoregressive (AR) model used to express the expected value of one or multiple variables based on times-series analysis. The lagged values for the explanatory variables are used in the model. The main advantage of these type of modeling, applicable to forecasting problems, is that they *a priori* require no knowledge about the interdependencies and influences between variables in the model. By contrast, the DICE is a model were each relationship between variables need to be *a priori* expressed before running the model. The VAR models are often used in the field of finance because of they strong empirical applications. Concretely, this type of models usually lead to this form of equation:

$$y(t) = \sum_{i=1}^{p} A_i y(t-i) + \varepsilon(t)$$

where $y(t)$ is the vector of variable we wish to track, $\varepsilon(t)$ is an error term, $p(t)$ is a lag parameter and $A$ is a matrix representing the influence of the lagged values of the variables in the vector $y$ on its value at $t$. These models, similarly as the ones with a fix a parametrized structure like the DICE, are blind to any disruptive change.

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# Cross asset

INVESTMENT STRATEGY

April 2019 | Working paper

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